UNIVERSIDADE ESTADUAL DE CAMPINAS

INSTITUTO DE ECONOMIA

ELIAS YOUSSEF HADDAD NETTO

Stylized Facts and Industrial Dynamics: An empirical analysis of Brazilian Manufacturing (1996-2013)

Fatos Estilizados e Dinâmica Industrial: uma análise da Manufatura Brasileira (1996-2013)

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Profª. Dr.ª Ivette Raymunda Luna Huamaní – orientadora

Dissertação de Mestrado apresentada ao Programa de Pós-Graduação em Ciências Econômicas da Universidade Estadual de Campinas para obtenção do título de Mestre em Ciências Econômicas.

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INSTITUTO DE ECONOMIA

ELIAS YOUSSEF HADDAD NETTO

Stylized Facts in a Complex Environment: An Empirical Analysis of Size, Growth and Productivity for Brazilian Firms

Fatos Estilizados num Ambiente Complexo: Uma Analise Empírica de Tamanho, Crescimento e Produtividade para Firmas Brasileiras

Profª. Dr.ª Ivette Raymunda Luna Huamani – orientadora

Defendida em 05/12/2017

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A Ata de Defesa, assinada pelos membros da Comissão Examinadora, consta no processo de vida acadêmica do aluno.

This dissertation is dedicated to the memory of Herbert Simon.

"Economics consists of theoretical laws which nobody has verified and of empirical laws which nobody can explain."

Kalecki, as quoted by Steindl (1965, p. 18).

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Abstract

The growing availability of longitudinal data at the firm and plant levels in the last decades has enabled a series of advancements in what is known about stylized facts in Industrial Organization, especially in the field of evolutionary microeconomics. Some of these industrial patterns constitute one of the oldest regularities in Economics, such as the Gibrat Law of Proportionate Effect, and the lognormal and Pareto fits of firm size distributions. However, there is a gap in the literature regarding how the same phenomena behave in developing countries. This work aims to contribute to this literature by providing original results using firm-level data from Brazil's Industrial Survey (PIA). Particularly, we want to understand better how well those stylized facts found in developed countries describe productive structures of peripheral economies such as Brazil. Then, we are going to inspect if these facts are persistent in time and robust to disaggregation. Finally, we are going to contrast these results with current theoretical views about market characteristics, competition, firm's capabilities and growth. To achieve this, we analyze the most important proxies for firm size, productivity, growth and productivity change, and go further with exercises of concentration, distribution probabilities and, finally, their parametric fitting. Also, we investigate the recent venue of research pertaining market selection. For this, we perform decomposition exercises of the productivity change in its main components for PIA and Service Surveys (PAS), using different classes of firm size. Our contributions provided evidence of a ubiquitous heterogeneity in the most important metrics of size, growth and performance. There is also compelling evidence regarding the lognormal and Pareto shape of firm size distributions, which seems robust to disaggregation and persistent in time. Firm rates have a symmetrical shape, well described by an AEP distribution, with most tails at least Laplacian, which imply some kind of short-term correlation in the growth events. Finally, productivity distributions appear to have an asymmetrical shape, with some evidence of an "efficiency frontier" that limits the performance of the market leaders, while the left side of the distributions are mostly unconstrained and assume fatter tails. Regarding productivity decomposition for manufacture and service sectors, our results show that smaller firms for manufacturing appear to be much more affected by our proxy of competition or market selection, while for bigger firms in all industries, competition doesn't appear to "bite" as much as evolutionary theories would predict. Regarding the firm-specific internal variation, learning appears to be highly correlated to the economic cycle, and represents most of productivity change.

Keywords: industrial Organization; Firm Size Distributions; Stylized Facts; Market Selection.

Resumo

A crescente disponibilidade de dados longitudinais ao nível de firmas e plantas nas últimas decadas tem possibilitado uma série de avanços no que se sabe a respeito de fatos estilizados em Organização Industrial, especialmente no campo de microeconomia evolucionária. Alguns desses padrões constituem algumas das regularidades mais antigas em Economia, como a Lei de Gibrat, e o formato lognormal e de Pareto das distribuições de tamanho das firmas. Entretanto, há uma lacuna na literatura no que se refere ao comportamento desses mesmos fenômenos para países em desenvolvimento. Esse trabalho visa contribuir para essa literatura através de resultados originais para o Brasil utilizando de informação ao nível de firmas provindas da Pesquisa Industrial Anual (PIA). Particularmente, nós queremos entender quão bem os fatos estilizados encontrados para os países desenvolvidos descrevem estruturas produtivas de economias periféricas como o Brasil. Depois, iremos verificar se esses fatos são persistentes no tempo e robustos à desagregação. Finalmente, vamos contrastar esses resultados com visões teóricas a respeito das características dos mercados, competição, capacidade das firmas e crescimento. Para alcançar esses objetivos, nós iniciamos com uma análise das *proxies* mais importantes para tamanho das firmas, produtividade, crescimento e variação da produtividade, e avançamos com exercícios de concentração, distribuições de probabilidade e seu *fit* paramétrico. Além disso, investigamos a linha mais recente de pesquisa relacionada à seleção de mercado. Para tanto, fazemos decomposições da variação da produtividade nos seus componentes principais usando dados da PIA e da Pesquisa Anual de Serviços (PAS), para diferentes classes de tamanho. Nossas contribuições geram evidências de uma heterogeneidade ubíqua nas principais métricas econômicas. Também há evidência favorável para o formato lognormal e de Pareto para a distribuição de tamanho das firmas, ambos robustos à desagregação e persistentes no tempo. A distribuição das taxas de crescimento e variação da produtividade tem um formato relativamente simétrico, bem descritas por uma distribuição AEP, com a maior parte das caudas mais pesadas que uma Laplaciana, o que sugere algum tipo de correlação de curto prazo nas oportunidades de crescimento, i.e., o crescimento de uma firma não é independente do de suas competidoras. Finalmente, a distribuição da produtividade apresenta um formato claramente assimétrico, com evidência de uma "fronteira de eficiência" que limita a performance dos líderes do mercado, enquanto o lado esquerdo da distribuição é pouco constrangido, e assume caudas mais pesadas. Com relação a decomposição da produtividade para manufatura e serviços, nossos resultados demonstram que firmas pequenas da manufatura parecem ser muito mais afetadas por nossas *proxies* de competição, enquanto que, para firmas maiores de ambos os setores, a competição não parece ser tão agressiva quanto teorias evolucionárias prediriam. Em relação às mudanças internas das firmas, nossa métrica de aprendizado aparenta ser altamente correlacionada com o ciclo econômico, e representa a maior parte da variação da produtividade.

Keywords: Organização Industrial; Distribuição de Tamanho das Firmas; fatos estilizados; seleção de mercado.

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Introduction

The growing availability of longitudinal data at the firm and plant levels in the last thirty years has made possible a series of advancements in what is known about characteristic patterns for Industrial Organization, especially in the field of evolutionary microeconomics.

Some of these industrial patterns constitute one of the oldest regularities in Economics. Firms and markets have a few robust statistical properties in countries with wide discrepancies in their technological profiles, styles of competition and leading industries. As such, they were classified as “stylized facts”, and were mainly related to size, growth, productivity and market dynamics.

Among them, we highlight the ubiquitous heterogeneity for firms in different metrics such as productivity and size, regardless of the disaggregation level, sectoral specificities, time periods and geographical location (BARTELSMAN; DOMS, 2000; SYVERSON, 2011); the skewness in firm size distributions, closely approximated by a Pareto or lognormal distribution (AXTELL, 2001; DOSI *et al.*, 2008); the Laplacian shape of productivity and growth rates (STANLEY *et al.*, 1996; BOTTAZZI; SECCHI, 2003; BOTTAZZI *et al.*, 2007); and the weak link between productivity change and growth (BOTTAZZI *et al.*, 2010).

However, there is a gap in the literature regarding how the same phenomena behaves in developing countries. This work aims to contribute to this literature by providing original results for these metrics using firm-level data from Brazil’s Industrial Survey (PIA), and, more narrowly, the Service Survey (PAS).

Our main objective with this dissertation is twofold. First, we want to understand how universal are the stylized facts found in developed countries to describe enterprises for mid-income nations such as Brazil. Second, we are going to verify if these facts are persistent in time and robust to disaggregation. Finally, we are going to contrast these results with current theoretical views about market characteristics, competition, firm’s capabilities and growth.

Particularly, our results will shed some light over the following questions:

- Is there evidence of hierarchies, scale effects or an optimal size in firm size distributions? How well does the representative agent hypothesis fares against Brazilian data?

- Does market and customers interact in predictable ways? Does competition affect business opportunities and create characteristic shapes for firm growth distributions?

- How does productivity change? How markets compete? What is the role of learning and marketing selection? Does size matter?

In order to address these objectives, we perform a series of parametric and non-parametric statistical analyses. We begin with some descriptive statistics regarding the most important proxies for firm size, productivity, growth and productivity change, and go further with an analysis of concentration, distribution probabilities and, finally, their parametric fitting. For a broader perspective on the components of market dynamics, we perform a decomposition of productivity change in learning and selection for different classes of firm size.

The structure of this dissertation is divided in three chapters and a conclusion. In Chapter 1, we present a brief review of the most important stylized facts in Industrial Organization and their economic interpretation.

In Chapter 2, we analyze the structural facts related to the proxies of performance and size, such as labor productivity, total revenue and number of employees. The chapter starts giving some contextual information about Brazilian Manufacturing, its changes through the period and information regarding market concentration and heterogeneity. Finally, empirical distributions and their parametric fitting aim to evaluate the different hypotheses regarding the characteristic shape for Firm Size Distributions (FSDs), productivity and growth rates.

Chapter 3 investigates the recent venue of research pertaining market selection. This literature separates the components of productivity change in two main effects. The within effect represents firm-specific variations in productivity levels and is a proxy for learning processes that occur inside the firm, such as incremental or disruptive innovation. The between effect is the change in productivity due to market-share variation, and it is interpreted as a metric of competition. We did this exercise for different classes of firms size, both for Services and Manufacturing. In this chapter, we want to understand what are the main components of productivity change; how competition shapes the market, if at all; if there are some kind of selective pressure that improves the enterprises' performance, promoting the survival of the "fittest", as predicted in evolutionary models; and, finally, if firms learn through time, i.e., they improve their internal capabilities of generating more value with less resources. Also, it is important to evaluate these processes under the light of scale effects, since size may be relevant to influence such dynamics. Large firms have higher survival rates than smaller ones, and are more able to negatively impact the overall index for longer periods without exiting the panel, due to a sectoral crisis for example. Particularly for small firms, credit restriction limits the time available that low productivity firms have to catch-up with the market, requiring a steep learning curve.

The last chapter gives a summary with the major highlights from the research, some concluding remarks and venues for future investigations.

1 Stylized Facts and Industrial Dynamics

The purpose of this chapter is to make a narrative about the development of the research in Empirical Industrial Organization and to highlight the major stylized facts found so far. Accompanying each stylized fact, a possible economic interpretation is used as an example of how they provide a powerful way for understanding economic phenomena.

"Few if any economists seem to have realized the possibilities that such invariants hold out for the future of our science. (...) In particular, nobody seems to have realized that the hunt for, and interpretation of, invariants of this type might lay the foundations of an entirely novel type of theory." (SCHUMPETER, 1949).

In this chapter, we will present the stylized facts that are deemed as the most ubiquitous in Industrial Organization, being found in different sectors, countries and periods. Our particular line of research has among its great contributors Gibrat (1931) and Herbert Simon (SIMON; BONINI, 1958; IJIRI; SIMON, 1977). These two authors created most of the foundation of the field and set the research agenda for several decades (AUDRETSCH *et al.*, 2004).

The stylized facts that we are mainly interested are patterns found for firm sizes, growth and productivity and the ones connected to market dynamics, such as entry-exit and market selection. These are the most important proxies in economic analysis, and constitute a basic, generic description of both the market and the firms. It is also useful that these variables have a characteristic pattern, giving a hint of generic processes that may be underpinning the whole organization of the economy.

In this context, it is opportune to have a definition of a stylized fact. A stylized fact is some broad pattern or generalization that describes well some kind of phenomena or behavior most of the time, but which lacks the formal prescription of physical laws. As such, they may be rejected for particular periods or economies and yet still be a useful, meaningful representation of what an economy should look like. In other words, stylized facts are regularities found in the observed phenomena and now they are beggining to constitute the fundamental fabric of which theoretical models are made (DOSI *et al.*, 2017).

In economics, their importance is justified because they act as "rough rules" for the interpretation of reality, and simplify the analysis of chains of extremely complex economic events, whose interaction usually cannot be observed, either due to the lack of proper proxies or by data and measurement limitations. It was in this spirit that Kaldor (1961, pp.178) defined stylized facts:

> "Since facts, as recorded by statisticians, are always subject to numerous snags and qualifications, and for that reason are incapable of being accurately summarized, the theorist, in my view, should be free to start off with a "stylized" view of the facts – i.e. concentrate on broad tendencies, ignoring individual detail, and proceed on the "as if" method, i.e. construct a hypothesis that could account for these "stylized" facts, without necessarily committing himself on the historical accuracy, or sufficiency, of the facts or tendencies thus summarized."

Our interest with these stylized facts is to verify their universality. Universal patterns give us hope to have data-driven models of economic phenomena that are valid for diverse countries regardless of culture, business organization, industries or development level. These general models aim to capture what are the basic processes generating economic organization, and as such, may be proven as timeless phenomena.

Particularly, the stylized facts that will be reviewed in this chapter are 1) the Gibrat Law and the skewness of firm size distributions; 2) the Scaling Law; 3) the Pareto/Zipf Distribution; 4) the Laplacian shape of Rates and Productivity; 5) the ubiquitous heterogeneity of the markets, with occurrence of fat tails in most economic metrics and 6) the dynamics of market selection.

1.1 Firm Size Distributions

Gibrat and the Lognormal Shape

Robert Gibrat was one of the first researchers to find patterns in Industrial microdata. Using a sample of French firms in Manufacturing, Gibrat (1931) observed that 1) Firm Size Distributions (FSD) were very skewed and closely resembled a lognormal distribution; 2) the growth rates appeared to be uncorrelated with size, being well described by a random walk.

These two observations are strongly interlaced in the Gibrat hypothesis, also called Gibrat Law of Proportionate Effect. Based on his observations, Gibrat structured a model where firm growth is composed by numerous, uncorrelated and proportional random shocks. The model is given by a multiplicative process so that:

$$S_{i,t+1} = S_{i,t}(1 + c_{i,t}) \tag{1.1}$$

where $S_{i,t}$ represents a proxy for firm size i in period t, such as number of employees, total revenue or value added, and $c_{i,t}$ represents a random, independent shock[1]. Then, taking the natural logarithmic in both sides results in:

$$s_{i,t+1} = s_{i,t} + \log(1 + c_{i,t}) \tag{1.2}$$

or

$$s_{i,t+1} = s_{i,t} + e_{i,t} \tag{1.3}$$

which is the common presentation of Gibrat's Law. Then, growth rates $g_{i,t}$ are expected to be given by:

$$s_{i,t+1} - s_{i,t} = g_{i,t} = e_{i,t} \tag{1.4}$$

which means that they are basically uncorrelated and randomly distributed. To test the hypothesis, we estimate the regression[2]:

$$s_{i,t+1} = \alpha + \beta s_{i,t} + e_{i,t} \tag{1.5}$$

If the estimation of Equation (1.5) provides coefficients $\beta \neq 1$, then previous size is correlated with current size and Gibrat Law does not hold. Values higher than one imply that large firms grow faster, with a tendency toward monopolies, and values lower than one point to some reversal to the mean, with smaller firms growing faster. In practice, we are testing if size time series is given by a stationary process. When it is, firms are expected to converge for a finite size, and when it is not, firms that are bigger will grow more than smaller ones, thus culminating in monopolies.

The model basically represents growth rates as a random walk for $\beta = 1$ and, when the number of shocks is large enough, it has the nice property to produce a lognormal distribution for firm sizes[3]. These two characteristics fills the observations made originally by Gibrat.

A lognormal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. This means that if X is lognormally distributed, then $Y = log(X)$ follows a normal distribution. Figure 1 shows lognormal density distributions with different parameters of a random variable X, and the respective normal distributions of $log(X)$. Also, we show both the cumulative distribution function (CDF), which represents the probability of a random value Y being smaller than a particular

[1] The particular distribution of the shocks does not matter, but several models now incorporate the idea of a Laplacian distribution for the yearly growth rate, which will be reviewed in the next sections.

[2] There are several ways to specify the model to correct for specific problems, such as heterokedasticity and autocorrelation in the growth rates. A discussion of these issues is available at Lotti *et al.* (2003).

[3] This is a direct consequence of the Laplace-Liapounoff Central Limit Theorem, which requires random shocks much smaller than one and the number of shocks to be large. A demonstration is available in Kalecki (1945) and Steindl (1965).

threshold, and the log-complementary distribution function (CCDF), which represents a survival function in a double log-scale, and is especially useful to analyze the right tail of the distribution.

The probability density function of a lognormal distribution is given by:

$$f(x|\mu,\sigma) = \frac{1}{x}.\frac{1}{\sigma\sqrt{2}}\exp\left(-\frac{(\ln x-\mu)^2}{2\sigma^2}\right) \tag{1.6}$$

where μ represents a location parameter and σ is a scale parameter for the respective normal distribution.

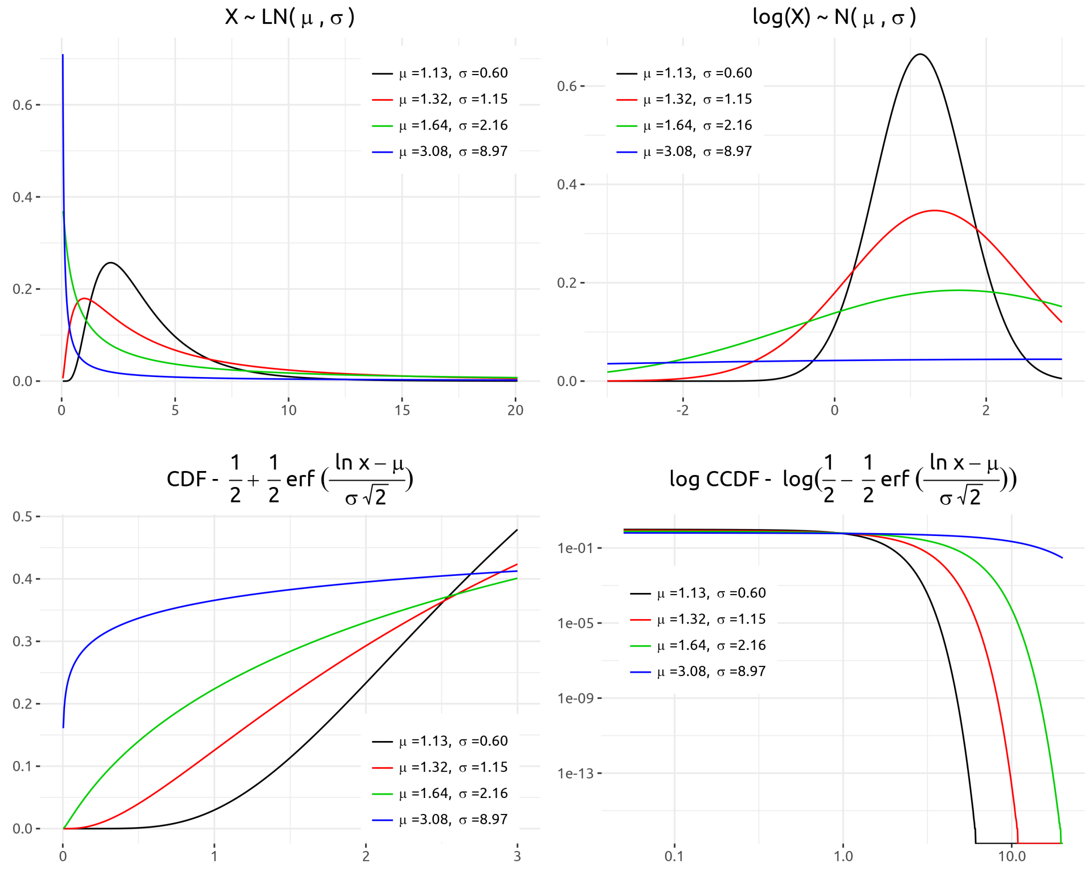


Figure 1 – Different visualizations of a variable X following a lognormal distribution. The function erf refers to the error function.

Gibrat hypothesis was tested extensively for other countries[4] and, according to Mansfield (1962), it has at least three different interpretations: 1) the law holds for exiting and surviving firms; 2) the law holds only for surviving firms; 3) the law only holds for firms that are larger than some threshold of efficient scale (SIMON; BONINI, 1958). In general, Gibrat's Law is found to hold as a very good first approximation to empirical data, and finds favorable support when using samples with only larger firms (HART; PRAIS,

[4] In-depth reviews are available at Geroski (1995), Sutton (1997) and Audretsch *et al.* (2004).

1956; SIMON; BONINI, 1958; HYMER; PASHIGIAN, 1962; GEROSKI; MACHIN, 1993; LOTTI *et al.*, 2001; LOTTI *et al.*, 2003).

However, with broader samples in general the law is rejected, since a correlation between size and growth appears with the inclusion of smaller firms (SAMUELS, 1965; PRAIS, 1976; EVANS, 1987a; EVANS, 1987b; HALL, 1987; DUNNE *et al.*, 1988; DUNNE *et al.*, 1989; REID, 1995; AUDRETSCH *et al.*, 1999; M.; NERIINGER, 2000). The inclusion of smaller firms pulls the coefficient below unity, and bigger firms appear to grow slower than smaller ones. Age also seems to matter, as younger firms grow faster but have less probability of survival (CABRAL; MATA, 2003).

Nowadays, rather than a precise description of reality, Gibrat is seen as a benchmark for other models. Still, the skewness in firm size distributions is a stylized fact in itself, and has been found for different countries, such as UK, US, France, and Italy, , with lognormal distributions usually describing well the aggregate data (HART; PRAIS, 1956; SIMON; BONINI, 1958; HYMER; PASHIGIAN, 1962; BOTTAZZI; SECCHI, 2003; BOTTAZZI; SECCHI, 2005; DOSI, 2005; BOTTAZZI *et al.*, 2007; DOSI *et al.*, 2008).

The Scaling Law

Besides the uncertainty regarding the Gibrat hypothesis, its most important contribution was to catalyze a process of empirical reasoning in economics. Based on the hypothesis, Kalecki (1945) observed that if size followed a random walk, the standard deviation of the proxy used for size should increase continuously, a fact that is not empirically verified. So as a correction mechanism, he proposed that the variance of growth should decrease linearly with size. This would make the variance of the distribution of firm size stable. In this way, he proposed what is now known as the Scaling Law.

The argument was extended by Hymer and Pashigian (1962). If large firms are a random sample of independent small plants that follow a distribution with the same mean and variance, it is possible to demonstrate that, by the theorem of the standard error of the mean, the variance of size should decrease exponentially by a factor of $1/2$[5].

The Scaling Law is usually estimated using a binned regression and can be described as:

$$\sigma(g_j) = \alpha + \gamma s_j + e_j \tag{1.7}$$

where $\sigma(g_j)$ represents the standard deviation of growth rates of firm measured as the difference of the natural logarithms of size for two consecutive periods t binned by size in the class j, and s_j represents the average of the natural logarithm of the proxy used for firm size, binned in class j. Beyond Hymer and Pashigian (1962), Mansfield (1962) and

[5] The demonstration is available at Hymer and Pashigian (1962), and in Buldyrev *et al.* (1997) in a more rigorous fashion

Singh and Whittington (1975) appears to be the first to test the hypothesis on empirical data, using US and UK data, respectively. More recently, it was also tested for broader US database (STANLEY *et al.*, 1996; AMARAL *et al.*, 1997; L. Amaral *et al.*, 1997; BOTTAZZI; SECCHI, 2003), Italy (BOTTAZZI *et al.*, 2007) and the International Pharmaceutical Industry (BOTTAZZI; SECCHI, 2005). In general, the results support the Law for US and the International Pharmaceutical Industry, with a coefficient of around -0.2. A puzzling exception is the Italian Manufacturing Sector, where no correlation whatsoever was found.

The law asserts that the variability of growth rates should decrease with firm size, implying that bigger firms would grow in relatively more uniform rates. In other words, this would create a mild convergence of growth rates as firms get bigger. Since these firms have most of the market-share (as seen previously due to the high right-skewness of size distributions), they would grow close to an overall economy rate. This led (BOTTAZZI; SECCHI, 2006b) to argue that this convergence is due to product differentiation. They create a model where bigger firms grow by diversifying their portfolio. Since they operate in more sub-markets as they get bigger, the variance of growth rates in relation to size decreases.

The Pareto Shape

The Gibrat and Scaling Laws were the basis of the research for empirical distributions of firm level variables. Since the distributions for size are strongly right-skewed, a lognormal fits very well the aggregated data. However, it was observed that the empirical data departed increasingly from a lognormal as one gets closer to the right tail of the distribution. This tail seems better represented by a Pareto Distribution.

The Pareto Distribution was already found to describe well the size of cities, exporters value, number of word occurrences in a book and the number of citations of research papers (NEWMAN, 2005). Other examples of Power Laws are the fall of production costs, which appears for, among others goods, microchips, aircraft manufacturing and light bulbs[6]. Income and wealth distribution also are well fitted by power laws, with tail exponents between 1.5 and 3 (ATKINSON; PIKETTY, 2007).

The Pareto Distribution appears first in the work of Vilfred Pareto (1896) when studying the upper tail of the income distribution. The complementary cumulative distribution function (CCDF), also called a survival function, of the Pareto distribution can be described as:

$$Pr[S \geq s_i] = (\frac{s_{min}}{s_i})^{\alpha} \tag{1.8}$$

[6] For a extensive bibliography see Dosi *et al.* (2010, p. 71).

where $Pr[S \geq s_i]$ represents the probability of a random value S to be higher than some particular threshold s_i; s_{min} and s_i represents, respectively, the smallest observation[7] and the value of the i observation measured by a proxy of size and α is a parameter representing the velocity of decay of the distribution. Basically, for FSDs, the law asserts that the probability of finding a firm greater than s_i is inversely proportional to the value itself, which characterizes a Power Law[8]. For example, if $\alpha = 1$ and $x_{min} = 1$, the probability of finding a firm higher than $s_i = 10$ is 10%, while if $s_i = 100$, the probability is 1%. Calling $Pr[S \geq s_i] = R(s_i)$, and taking the natural logarithm on both sides generates:

$$\log(R(S_i)) = \alpha \log(S_0) - \alpha \log(S_i) \tag{1.9}$$

which represents the log-complementary cumulative distribution function, which has the useful property of being simply described by a linear function. Thus, a common representation of the Pareto distribution is to plot its CCDF in a log-log scale. Figure 2 shows the density, cumulative and survival function of the Pareto Distribution. When α equals 1, this power-law reduces to the so called Zipf Law[9].

The importance of power laws in nature and economics cannot be overstated. One of their advantages is that they are independent of unit of measurement. This scale-free property is useful for comparisons with different sources of data, or even variables. As Gabaix (2009, pp. 257) said: "Power Laws give the hope of robust, detail-independent economic laws"[10]. Another advantage is that they can be used as a type of "coarse-grained" model of real phenomena[11]. Additionally, given that the α coefficient is a measure of the inclination of the CCDF, it is a useful proxy of market concentration, since the lower the α, the fatter the tail will be.

Finally, power laws were successfully used to represent distributions with extreme events, such as stock market prices' variation. They are able to approximate well the body and the tails of the distributions. In this way, they constitute a theoretical ramification that is able to explain not only the common market behavior, but also its crashes (MANDELBROT, 1963; GABAIX, 2009), which goes against the traditional way of ex-

[7] The Pareto distribution is only defined with a cutoff point, i.e, there must be some minimum value for the distribution. That is why it is usually stated that it only describes the right-tail after some threshold.

[8] A review of the usage of power laws in economics in available in Gabaix (2009).

[9] Zipf Law is a special case of the Pareto Distribution. It was discovered by Zipf (1949) in the study of word usage in different languages, and was one of the first observations of power laws in probability distributions. For simplicity, Pareto and Zipf will be treated indistinctly in this chapter. In a sense, Zipf and Pareto also provide alternative visualizations. Zipf Law usually is presented with a rank-frequency plot, with the probability in the vertical axis. Pareto did the other way around, with the proxy for size in the vertical axis (NEWMAN, 2005). A third alternative is to present not the rank-probability, but the rank itself (STANLEY *et al.*, 1995) or its logarithmic (DOSI *et al.*, 2008). This has create some confusion in the literature, but all of them are representing basically the same information.

[10] Santa Fe Institute today is the leading organization in this venue of research (AXTELL, 2001; WEST, 2017).

[11] Coarse-grained modeling is a way of simulating the behavior of complex systems using their simplified representation. For a recent example in Social Sciences, see Zou *et al.* (2012).

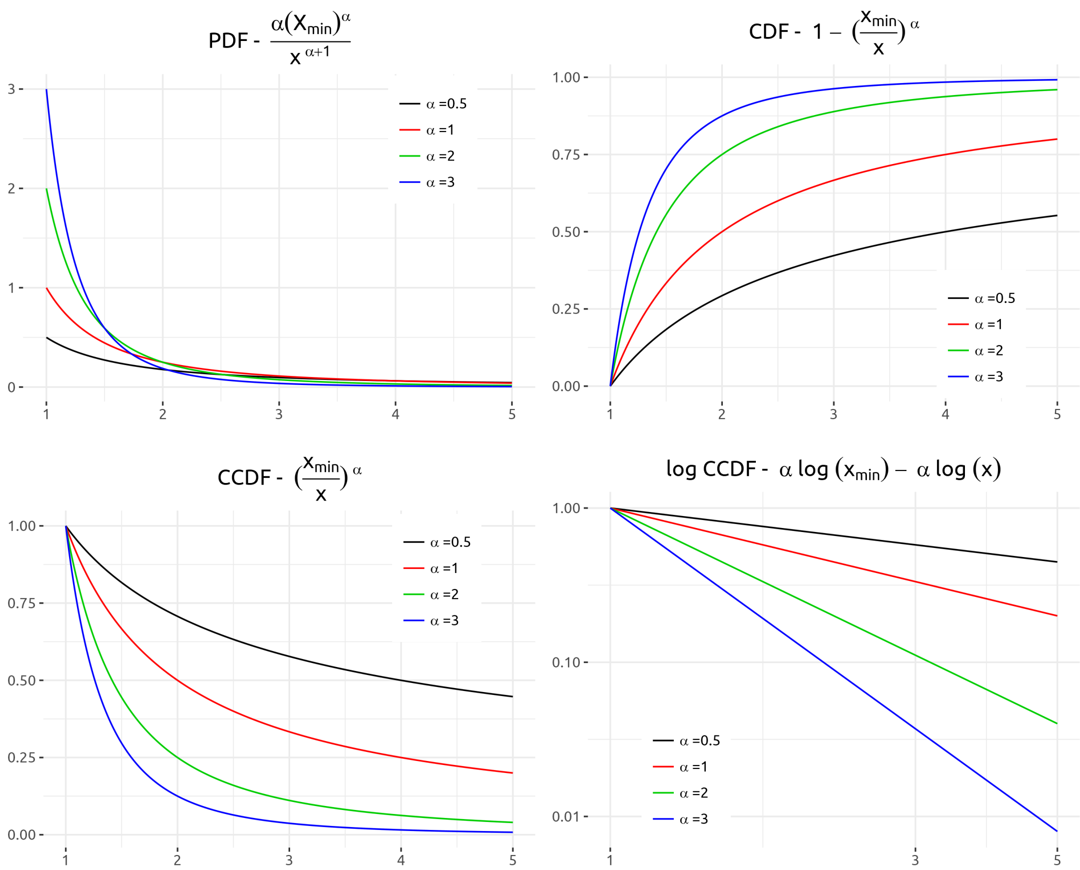


Figure 2 – Pareto Distribution with different parameters.

pressing price fluctuations as Gaussian random walks, with extreme events being classified as "outliers" (STANLEY *et al.*, 2007).

The first to elaborate stochastic processes culminating in the Pareto Distribution was Yule (1925), who described the distribution of biological genera by number of species. In economics, Champernowne (1953) was the pioneer, elaborating a model to explain the Pareto format of the income distribution using stochastic shocks.

Later, Herbert Simon observed that the process used by Champernowne was basically the same as Yule's (SIMON, 1955) and for which he proposed another version, in the context of firm size distributions (SIMON; BONINI, 1958). The model needed the introduction of some small frictions in the Gibrat Law of Proportionate Effect to modify its convergence to a Pareto distribution, instead of a lognormal distribution. So, besides the law of proportionate effect (i.e, that expected growth is uncorrelated with size), it was necessary that market growth produced a constant rate of entry of new firms (i.e., a constant probability of growth in the market being caused by new entrants). This generated a Yule distribution, which closely approximates a Pareto distribution in the upper tail. The main problem with this model was that convergence from a sample of identical firms was really slow (KRUGMAN, 1996a), and the market average growth needed to be really substantial to achieve a smooth distribution in a reasonable time-frame[12].

Other issue is the relation between growth rates' variance and average size, i.e, the Scaling Law. In the model of 1958 (SIMON; BONINI, 1958), a more stringent version of Gibrat Law was assumed, which rejects a correlation between the variance of growth rates and size. But, as the recent venue of studies regarding the Scaling Law showed, not only the variance of the shocks decrease as the firms become larger, there are signs of correlation among the growth of different plants of the same firm (since the empirical γ is smaller than 1/2) , thus leading to evidence of scale gains or common managerial practices in multi-plant firms (HYMER; PASHIGIAN, 1964; SIMON, 1964; STANLEY *et al.*, 1996; BOTTAZZI *et al.*, 2007; GABAIX, 2009), what makes the model of 1955 more accurate (SIMON, 1955). As such, other variations were proposed for the mechanisms used to generate a Pareto distribution, such as Luttmer (2007), where technology available to new entrants determined the growth rate of the economy[13].

For FSDs, the Pareto shape is found to describe very well the aggregate distribution for several countries with different proxies (AXTELL, 2001; DOSI, 2005). There are, however, some caveats. Usually, when going to finer levels of disaggregation, sectoral discrepancies and the occurrence of multimodalities in the empirical distributions led some authors to question whether or not both the Pareto and lognormal shape wouldn't be an outcome of the sheer aggregation of different sectors, diminishing their importance as stylized facts (DOSI *et al.*, 1995; BOTTAZZI; SECCHI, 2003; BOTTAZZI *et al.*, 2007).

Also, beggining in the 1970's, the availability of new data made apparent the existence of a small concavity in CCDF distribution of firm sizes, which weights favorably to the lognormal hypothesis (see Figure 1). This concavity is verified in recent works for Italian, and especially, French firms (DOSI *et al.*, 2008). Ijiri and Simon (1974) stipulate two possible explanations. The first was to adapt a previous model generating a correlation among firm growth rates with a decaying impact, where older growth episodes affect less current growth than newer episodes. The second was related to mergers and acquisitions. External growth due to mergers would contribute noticeably to increase concentration in middle-size firms, without affecting small and very large firms. Small firms, with minimal exceptions (such as startups), are not the focus of M&A. Very large firms usually are forbidden to enter into such processes due to anti-trust laws. Both mechanisms would create a small concavity in the distribution while preserving the linear decaying in the tail.

Overall, the agreement between the empirical values and the parametric fits are very high, making both the lognormal and the Pareto Distribution very good descriptions of the data. The explained variance of the log-rank estimation of the Pareto distribution is

[12] An exposition of the model of Simon is beyond the scope of this review, but the avid reader will found a less cryptic version in Krugman (1996b) and variants in Steindl (1965) and Marsili (2005).

[13] The mechanism used in this paper is close to the one exposed by Gabaix (1999) for the Zipf distribution of city sizes.

usually above 90% (AXTELL, 2001; MARSILI, 2005). It must be noted, however, that the explained variance is not a good test to verify the model validity or to differentiate if the true distribution is a Pareto or a lognormal (CLAUSET *et al.*, 2009).

In practice, to differentiate between the lognormal and the Pareto distribution for the tail distribution of firm sizes may not be possible with a finite sample (CLAUSET *et al.*, 2009). Still, there is ongoing research and debate trying to verify if FSDs tails are in fact Pareto/Zipf or lognormal (CROSATO; GANUGI, 2007; BEE *et al.*, 2013) and if this stylized fact is robust to disaggregaton (MARSILI, 2005). And while it may be useful to know the asymptotic distribution to better define the mechanism generating the FSD format in any theoretical model, it should be noted that given the difficulty in determining the true distribution, the theorist has some liberty in defining which mechanism one finds most convenient, while respecting the other stylized facts (e.g., the Laplacian shape of firm growth rates - see next section).

1.2 Rates and Productivity Distributions

The Laplacian Shape

This type of exercises were extended to the analysis of firm growth, productivity and productivity change. The work of Stanley *et al.* (1996) discovered a very stable pattern for firm growth rates for US Manufacturing.

Particularly, probability densities of growth rates in a log-lin scale show a very stable "tent-shape", that can be well approximated by a Laplace Distribution, a function characterized by its fat tails (see Figure 3). Their work was followed by Bottazzi and Secchi (2003), Bottazzi and Secchi (2005) and Bottazzi *et al.* (2007), finding similar patterns for the International Drug Industry and the Italian Manufacturing.

The probability density function of a Laplace distribution is given by:

$$f(x|u,\alpha) = \frac{1}{2\alpha}\exp\left(-\frac{|x-\mu|}{\alpha}\right) \tag{1.10}$$

where μ represents a location parameter and $\alpha > 0$ is a scale parameter.

This result is highly counter-intuitive since Gibrat's model doesn't expect any characteristic format for the short-term growth, while predicting, by the Central Limit Theorem, a normal growth rate distribution for the long term. Basically, since the shocks are expected to be uncorrelated, their multiplication translates in a sum of logarithms of independent random variables, which, since they have a finite variance, converge to a normal distribution[14]. The fact that these shocks don't generate a normal distribution in

[14] To be more precise, the cumulative distribution of shocks $(1 + e_{i,t=1})(1 + e_{i,t=2})...(1 + e_{i,t=n})$ with $(1 + e) \geq 0$ should appear as a lognormal distribution while $\log(1 + e_{i,t=1}) + \log(1 + e_{i,t=2}) + ... + \log(1 + e_{i,t=n})$, or the rates themselves, should converge to a normal distribution.

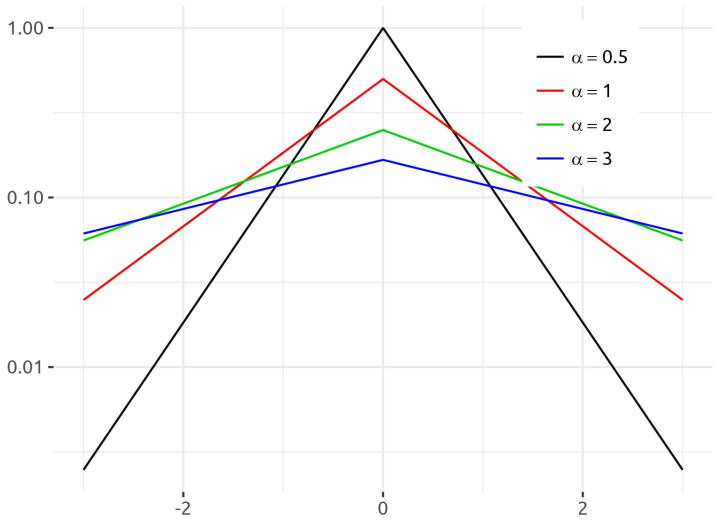


Figure 3 – Laplace Distribution with different scale parameters and $\mu = 0$. The vertical axis is in log scale.

the short term imply the existence of some underlying correlation mechanism. Also, as showed by Stanley *et al.* (1996), Amaral *et al.* (1997) and Bottazzi and Secchi (2006a), this correlation seems to last longer than one-year periods, and even with seven years, the distribution is still far from normal.

This correlation was modeled by Bottazzi and Secchi (2006a) in the Simon tradition of "islands of opportunity" (IJIRI; SIMON, 1977). Basically, the model makes the assumption of a finite set of preexisting growth opportunities in the short-term, and aims to simulate the firms' competition for these scarce possibilities. By introducing a self-reinforcing mechanism, where firms that won in the past have higher probability of winning in the future and making the distribution of business opportunities to follow a Bose-Einstein distribution, they are able to faithfully recreate the Laplacian nature of firm growth rates.

On the empirical side, a more recent family of distributions, apt to adjust each tail of the distribution independently, was introduced by Bottazzi and Secchi (2011). The Asymmetric Exponential Power (AEP) distributions[15] were later applied to growth, productivity and productivity change, for countries such as Italy (BOTTAZZI *et al.*, 2010), India (MATHEW, 2017) and China (YU *et al.*, 2015b).

Overall, growth and productivity change rates distributions present a remarkably symmetrical shape, stable over time and robust to disaggregation. These distributions are well fitted by symmetrical Laplacian distributions. The productivity distributions, on the other hand, are asymmetric, with the left side close to a Laplace distribution, and the right side better approximated by a normal one.

[15] The AEP distributions will be formally presented in Chapter 2.

1.3 Market Selection and Heterogeneity

Market selection

The relation between growth and productivity is given by different families of theoretical models, and usually involves more productive firms gaining market-share either by lowing mark-up or through larger investments driving more innovation and better products. A first approach is given by what was called an "evolving equilibrium" or "dynamic equilibrium", and it was exposed in works that embed heterogeneity as a fundamental force, like Jovanovic (1982), Hopenhayn (1992), Ericson and Pakes (1995), Olley and Pakes (1996), Luttmer (2007) and Acemoglu *et al.* (2013). Another approach is given by the neo-schumpeterian literature, with the classic from Nelson and Winter (1982) and others like Winter (1984), Silverberg *et al.* (1988), Dosi *et al.* (1995), Silverberg and Verspagen (1995), Metcalfe (1998), Bottazzi *et al.* (2001), Winter *et al.* (2000), and Winter *et al.* (2003).

Furthermore, there are other theoretical works in which this relationship plays a central role: in determining the evolution of routines, such as in a Generalized Darwinism perspective (ANDERSEN, 2004; HODGSON; KNUDSEN, 2004); in neo-Schumpeterian models, where it appears as a mathematical expression for the construction of evolutionary explanations in line with the replicator's dynamics (METCALFE, 1994; METCALFE, 1998; METCALFE; RAMLOGAN, 2006) and in the general principle of selection of evolving systems (KNUDSEN, 2004).

Besides the topic of productivity being extensively explored throughout the twentieth century[16], the first studies using modern micro-level data appeared only in the early nineties. Baily *et al.* (1992) was one of the pioneers to describe the relationship between productivity and market composition for the US Manufacturing.

An important turning point on the discipline was the growing availability of micro-level data with a systematic representation of industry at the firm level. This led to numerous studies evaluating the transformation of productivity using decomposition methodologies and parametric estimations. Among them, methods frequently used in the literature are the modified version of Baily *et al.* (1992), proposed by Foster *et al.* (2001), Griliches and Regev (1995) and the Price Equation (HOLM, 2010; LUNA *et al.*, 2015).

Regarding productivity decomposition, these exercises usually decomposes productivity change in four components. The within effect represents firm-specific variations in productivity levels, and is a proxy for learning processes that occur inside the firm, such as incremental or disruptive innovation and learning by doing. The between effect is the change in productivity due to market-share variation, and it is a metric of competition or selective pressure acting to promote the *fitness* of the market. The two other effects are the entry and exit dynamics, where they provide proxies for entry barriers, entrepreneur-

16 Salter (1966) is an earlier example of the kind of analysis conducted here.

ship (as in the Schumpeter Mark I regime) and hardcore selection caused by the *death* or exiting of firms from a sector. These works were conducted for several countries, like Israel (GRILICHES; REGEV, 1995), United Kingdom (DISNEY *et al.*, 2003a; DISNEY *et al.*, 2003b), Germany (CANTNER; KRUGER, 2008), Chile (PETRIN; LEVINSOHN, 2012) and Canada (BALDWIN; GU, 2006).[17]

In general, the studies find a minor role for the selection effect - the reallocation of shares between continuing firms[18] - with most of productivity change being caused by the movement of entry and exit of firms and due to internal variation. Parametric estimations of this process also corroborated these results. Dosi *et al.* (2015), improving on Bottazzi *et al.* (2010), found a small contribution of selection for France, Germany, UK and US, with most of the impact coming from the first difference of relative productivity - that is, the variation of the distance of each firm's productivity from the average productivity - rather than in relative productivity by itself, or the distance of each firm's productivity from the average. Analogous results are found in Chinese Manufacturing by Yu *et al.* (2015a).

Productivity, size and growth

Another important fact is the relationship of performance metrics with size. In general, even if the evidence is more dubious for smaller firms (LOTTI *et al.*, 2001), growth does not seem to be correlated with productivity or profitability (BOTTAZZI *et al.*, 2010; YU *et al.*, 2015a). On the other hand, size and productivity are important metrics for survival, where smaller and less productive firms die faster (BAILY *et al.*, 1992; GRILICHES; REGEV, 1995), a phenomenon that seems to be related with age (CABRAL; MATA, 2003). The fact, however, that most studies on the subject don't have have access to the true age of firms[19] severely limits the knowledge of the real impact of this variable. Finally, entry and exit seem to be highly correlated, with sectors with a high number of entrants usually having a high number of exiters. That is, markets seems to be relatively stable in size, at least for UK (DISNEY *et al.*, 2003a).

Heterogeneity

Among the empirical results for firm productivity we have a great heterogeneity found regardless of the level of disaggregation (BAILY *et al.*, 1996) and its high persistence through time (BARTELSMAN; DHRYMES, 1998), with fat tails and a significant intra-sectoral dispersion, which does not vanish in finer levels of disaggregation (GRILICHES;

[17] For two reviews of the literature see Bartelsman and Doms (2000) and Foster *et al.* (2001).

[18] Some studies, like Disney *et al.* (2003a), even find a negative value for the between component, suggesting a reallocation to less productive firms.

[19] Usually, the researchers know the existence of the firm only by its presence in the data panel, where its omission in particular years does not tell its fate, e.g., bankruptcy or exit due to the minimum size threshold of the survey that produced the data.

REGEV, 1995; BOTTAZZI *et al.*, 2007; BOTTAZZI; SECCHI, 2003; BOTTAZZI; SECCHI, 2005; YU *et al.*, 2015b). This occurrence is not restricted to productivity, but in fact affects most economic metrics, and the previous sections highlight the existence of fat tails for growth rates and diverse metrics of size, which also demonstrated high skewness, regularities that were found in different countries, time periods and sectors.

In fact, the ubiquitous nature of heterogeneity was vividly described by Griliches and Mairesse (1997):

> "we (...) thought that one could reduce heterogeneity by going down from general mixtures as 'total manufacturing' to something more coherent, such as 'petroleum refining' or 'the manufacture of cement'. But something like Mandelbrot's fractal phenomenon seems to be at work here also: the observed variability-heterogeneity does not really decline as we cut our data finer and finer. There is a sense in which different bakeries are just as much different from each other as the steel industry is from the machinery industry."

1.4 Conclusion

This chapter presented several stylized facts found in microlevel firm data. Among them, we would like to highlight: a) the Pareto and lognormal shape of Firm Size Distributions; b) the Laplacian Shape of productivity, growth rates and productivity change; c) the ubiquitous heterogeneity found in most economic metrics; d) the low selective pressure of the markets, with the most important components of productivity change being the firm-specific variation and entry-exit dynamics with the constant turn-over of firms; e) the weak relationship between productivity and profitability with growth.

These stylized facts appeared in different countries and periods, and constitute a benchmark that any theoretical model of industrial phenomena should aim to pass.

2 A survey on Stylized Facts for Brazilian Manufacturing: 1996-2013

International studies found important patterns in Industrial Organization regarding market characteristics, such as a widespread heterogeneity in economic metrics, the lognormal/Pareto distribution of firm sizes and the Laplacian distribution of firm growth rates. However, there are few studies testing the validity of these patterns for developing countries. The main focus of this chapter is to fill this gap for Brazilian Manufacturing. Using the microdata from the Brazilian Industrial Survey from 1996 to 2013, we check the robustness of these patterns under different proxies for firm size, growth, productivity and concentration, and for different levels of disagreggation. Our results suggest that, despite the significant differences among individual sectors, there is a core set of regularities that seems to hold for all of them, such as the lognormal/Pareto shape of Firm Size Distributions and the Laplacian shape of firm growth rates. Evidence for Brazil corroborates the results found for developed countries. These stylized facts, then, may describe ubiquitous processes driving market organization in economics.

JEL: C14, D22, L11, L60.

This chapter investigates statistical properties of Manufacturing firms related to size, concentration, productivity, growth and their seemingly ubiquitous heterogeneity, measured by a variety of proxies. Our main objective is to see which of the most common stylized facts explored in Chapter 1 are empirically supported for Brazil. Specifically, we are going to ascertain if 1) Brazilian firms are characterized by the same large skewness and wide dispersion in most economic metrics as found for other countries; 2) to see whether there is any pattern in markets concentration; 3) if there is favorable evidence for the Pareto or lognormal shape of firm size distributions; and 4) if there is favorable evidence for the Laplacian shape of firm growth rates and productivity variation distributions.

The analysis is performed for a panel of 467.695 observations over 1996-2013 from the Brazilian Industrial Survey (*PIA Empresa* Survey).

This work provides compelling evidence against some common hypothesis in economics. First, does any notion of an optimal size or representative agent portrait a good representation of the Brazilian Economy? No, the evidence available shows a wide heterogeneity in all metrics, robust under any level of disaggregation and persistent over time.

Second, is sectoral heterogeneity in performance metrics such as productivity related to growth differentials? Not necessarily. Notwithstanding the significant intersectoral differences in productivity and average growth rates among sectors, the distributions of growth rates have a fairly similar shape.

Third, does the notion of incremental innovation and technological trajectories as being smooth most of the time translate themselves in smooth growth or productivity change? Again, no, with extreme deviations and fat tails being characteristic not only of growth rates, but also of most economic metrics.

In order to show our investigation, the rest of this work is divided in three main lines, described below.

The first line gives some contextual information about Brazilian Manufacturing and its evolution through the period 1996-2013.

The second analysis explores the market concentration of Brazilian Firms. Concentration indexes were the basis of the discussion for the introduction of Pareto Distributions in economics, first with Pareto (1896) studying wealth distribution, then in the Industrial Sector with Champernowne (1953) and Simon and Bonini (1958), which creates a natural bridge between this and the next topic. We measure the right tail concentration of the top 4 firms over the top 20 to see if there is an "unequal" division of the shares between the leaders over profits, revenues and workers, and to verify if there is any tendency to the shrinkage of this difference over time. As a country with large income inequalities, one would expect to see a large market concentration as well, which is still reminiscent of the way production evolved in Brazil, composed by state-owned monopolies mixed with subsidiaries of large multinational groups.

The third line of investigation constitutes the core of this work, and deals with probability distributions and their moments. We perform aggregated and disagreggated estimations for size, growth and productivity distributions in cross-sectional and yearly views. Since Gibrat (1931) stated the Law of Proportionate Effect, i.e., that firm growth appears to be uncorrelated with its size, patterns in distributions were found regarding the market structure and organization for a broad range of countries and metrics. There is evidence of skewed distributions for firm size, closely approximated by a Pareto or lognormal distribution, at least in the aggregated level (HART; PRAIS, 1956; IJIRI; SIMON, 1977; STANLEY *et al.*, 1995; AXTELL, 2001; CABRAL; MATA, 2003), which usually extends over a wide support, implying the coexistence of firms with large differences in size, spanning several orders of magnitude.

A more recent strand of research regards the investigation of patterns for distribution of firm growth, productivity and productivity variation. Starting with Stanley *et al.* (1996), followed by Bottazzi and Secchi (2003), Bottazzi and Secchi (2005), Bottazzi *et al.* (2007), the Laplacian distribution, a curve characterized by its fat tails, seems to be well suited to describe the distribution of these metrics for countries as dissimilar as China, India, US and Italy (YU *et al.*, 2015b; MATHEW, 2017). The exponential decay of the Laplacian curve associated with their fat tails predicts extreme events as less infrequent as they would be if short-term events were completely uncorrelated„ as, for example, in a normal distribution.

Our results suggest that there is a wide heterogeneity in Brazilian firms, reaching similar outcomes as found in others works (NOGUEIRA *et al.*, 2014; ESTEVES, 2015; SQUEFF; NOGUEIRA, 2015), as evidenced by the moments of all variables, the concentration indexes and distributions. This brings us to the next question. In a scenario where firms have access to the same technology and workers, i.e., where there are no strong barriers to knowledge by means of patents or industrial secrets, it would not be expected such dissimilar performance. So why this happens?

A reason may be that, even inside the same market, enterprises face limitations to access the same suppliers, or the same price, and to reach the same number of costumers and markets. Scale may create priority and, with it, hierarchies. In other words, the wide intrasectoral deviation may produce evidence that it is not only the lack of technology or qualified personnel that limits the increase of firm productivity, but that they may also arise from a network with different roles to be fulfilled, and with them, different profitability levels (STURGEON, 2002). The idea that markets are intelligent and self-organized come as back as Hayek (1945). As the constraints of bounded rationality are as valid for enterprises as they are to people, signaling through hierarchy may be an important tool for market organization. Markets, then, rather than a "jungle", perhaps resemble more a king's court, where firms, as courtesans, compete but also cooperate, and have different *gravitas*.

Concentration indexes averaged between 35% and 50% in the tails, with little dispersion both among sectors and through time. This not only demonstrates that firms have different market powers inside the same sector, e.g. measured by number of employees, but that this market power produces different appropriability levels over the market results, and that such asymmetry is perpetuated over time, even when the leading firms are not always the same. Here, level of disaggregation matters. In this sense, the exercise may provide warnings from the usage of such indexes, as they may not be very precise about the market they are representing. In other words, since the market is itself composed by products, different levels of aggregation affects what the size of the market is, and thus, the market-share. Measures as proposed by Simon (IJIRI; SIMON, 1971), based on parametric distributions that describes the empirical data would be capable of, at least partially, circumvent this issue, especially when the parameters don't change significantly with finer levels of disaggregation.

For the distributions of firm size, we partly disagree with the conclusions presented by Bottazzi *et al.* (2007) in the sense that, whereas we do find that the apparently lognormal shape of firm size distributions may present multimodalities for some sectors or variables, this does not imply a poor performance for both lognormal and Pareto to describe the data, at least as a first approximation. In fact, the fit of the complementary cumulative distribution functions seems rather good, but we still need more formal tests to establish a preference of one distribution over the other, if at all. The overall shape is very robust to different sectors, periods and levels of disaggregation, with the same metrics sharing similar coefficients.

In relation to productivity distributions, their skewness and kurtosis, as well as AEP estimates[1] provide supporting evidence from what Dosi *et al.* (2012) calls an "efficiency frontier". Firms that are at the top of productivity in their sectors face constraints that are technological in their nature, which in turn create similar barriers to increasing productivity for all leaders.

Finally, growth and productivity change distributions display the same characteristic Laplacian shape found for other countries such as Italy, US, China and India (STANLEY *et al.*, 1996; BOTTAZZI; SECCHI, 2003; BOTTAZZI; SECCHI, 2005; BOTTAZZI *et al.*, 2007; YU *et al.*, 2015b; MATHEW, 2017). The distributions are heavy-tailed and fairly symmetrical, which may characterizes one of the most stylized facts in Empirical Industrial Organization. But, even if some attempted proposals tried to explain the type of mechanism that generates such distributions, as Bottazzi and Secchi (2006a), we still don't know much about their fine grain details nor do we have empirical evidence to support such processes.

At the same time, these distributions create an interesting contrast with some notions from innovation theory. If 1) we accept the concept of capabilities as a core set of practical knowledge, built slowly through a learning process, in the tradition established by Penrose (1959), and more recently by Gereffi *et al.* (2005), and 2) that this creates technological trajectories that are mostly subject to periods of incremental improvement with discontinuities following structural breaks due to radical or disruptive innovation (DOSI, 1982; DOSI; NELSON, 2010); then these rather smooth periods of incremental perfecting followed by large jumps of rapid change does not seem to affect the shape of growth or productivity variation rates distributions.

The rest of this work is divided as follows. The next section presents the data description and some context on Brazilian Manufacturing. The second section overviews the methodology used in this study. The third section presents the results and a discussion, and the fourth one finishes the paper with some highlights and a conclusion.

[1] The Asymmetric Exponential Power is a class of functions introduced by Bottazzi and Secchi (2011). They will be formally presented in the methodology section.

2.1 Data Description

Our analysis is based on the Brazilian Industrial Survey (PIA), which contains yearly census information for firms with more than 30 employees and in sectors with CNAE (National Classification of Economic Activities) codes between 5-33[2]. Our total sample comprehends 467.695 observations from 1996-2013 and the monetary values were deflated using 2-digit sectoral prices constructed with the GDP Implicit Deflator from the National Accounts[3]. Table 1 presents a brief description of the variables used.

Table 1 – Variables Description

Variable	IBGE CODE	Definition
Employes in 31/12	V004	Number of personnel employed in the last day of the calendar year.
Total Revenue	X13	Total Gross Revenue of sales, services and resale, plus financial revenues, commissions, licenses, non-operational revenues, assets variation, less returned sales and taxes.
Value added	X32	This is a modification of the original concept of value added in the sense that IBGE calculates only the Value Added in Industrial related activities. This is made by calculating the share of industrial products in gross revenues and multiplying this value by the net revenues plus the stocks variation and production for the firm's own assets (such as machines and etc) less the industrial operational costs.
Productivity	Calculated	Productivity is calculated as the Labor Productivity. It is given by X32/V004.
Productivity change	Calculated	This is the difference of the natural logs between the productivity of two consecutive years.
Growth rate	Calculated	This is the difference of the natural logs between the size of two consecutive years.

Source: PIA Publication / our elaboration.

Most of Brazilian Manufacturing firms are not captured by our subsample. In 1996, firms with up to 29 employees represented 76% of the number of firms in Manufacturing and Mining, or about 82.940 firms. In 2013, this number increased to 86% of the universe, or 296.154 firms[4]. However, according to IBGE (2013) they have a low share in number of employees (17% in 1996, 22% in 2013) and value added (6% in 1996, 8% in 2013). So,

[2] The split in sectors agree with the ISIC Rev. 04 Structure at the 2-Digit level, with minimal differences. Most expressively, alcohol production, which enters ISIC as a chemical product (Sector 20), is classified by IBGE's CNAE 2.0 as a biofuel (Sector 19), due to its extreme importance both as sole fuel and as a mixture with gasoline.

[3] The access to the data is restricted and due to privacy reasons we are committed to exclude any sector with less than 3 firms in any particular exercise. This makes some sectors, such as petroleum extraction, an activity that was a State monopoly until recently, to appear only in certain views. To avoid errors and fill-in mistakes, we also exclude firms with negative value added, negative total revenue, with less than 30 employees or that are registered as inactive.

[4] Part of this increase is due to IBGE starting to consider firms with less than 5 employees in the universe.

despite the importance that small firms have on the Brazilian economy and which our subsample ignores, it is important to remark that our database is responsible on average for about 80% of the employment and 90% of the value added in Manufacturing and Mining (SEBRAE, 2014).

Table 2 shows a summary with some statistics of size for the full data sample. The total values consolidate the results for each sector, in 1996 and 2013, respectively. It is interesting to see that most sectors increased their total sectoral values in all metrics, with best performers being metal ores (ISIC 7), refined petroleum (ISIC 19) and motor vehicles (ISIC 29). A few support activities of these industries clearly outdid themselves, as mining support (ISIC 9) and other transport (ISIC 30), which increased their revenue seventeen and six times in the period, respectively. Yet, these support activities are still too small to have a significant impact in the gross product. The worst performer is the tobacco industry (ISIC 12), which suffered with restrictive domestic policies regarding product design, marketing, and places allowed for consumption. The ranking is followed by leather (ISIC 15) and textiles (ISIC 13), which were object of heavy Chinese competition (SOARES; CASTILHO, 2016).

The firm averages for each sector tell a different story. For Total Manufacturing, all metrics decreased, with firms having less workers, revenue and value added. While the same best performers in the sectoral view maintain their gains at the firm level, most sectors experience a reduction in their metrics. ISIC codes 12 to 15, and 22 to 25 lost value in all three metrics, with group averages of −20% for number of workers, −36% for total revenue and −43% for value added.

This suggests that rather than an organic, internal growth, most sectors expanded due to the sheer increase in the number of firms. As can be noticed, most sectors almost doubled their number of enterprises with more than 30 employees. At the same time, as the firm averages went down, it is possible to infer that these entrants were lower in absolute values than their existing competitors for all size proxies. Figure 4 presents the number of firms for each sector through the whole period. As it is visible, with the exception of coal and lignite (ISIC 5), tobacco (ISIC 12) and wood manufacturing (ISIC 16), all sectors experience a steady increase in their populations.

Table 2 – Firm Size in Brazilian Manufacturing - 1996-2013

ISIC	Industry	Total Obs.		Number of Workers				Total Revenue				Value Added			
				Firm Avg.		Total Sector		Firm Avg.		Total Sector		Firm Avg.		Total Sector	
		1996	2013	1996	2013	1996	2013	1996	2013	1996	2013	1996	2013	1996	2013
5	Coal and lignite	12	12	322	474	4	6	82	112	1	1	54	53	1	1
6	Crude petroleum	NA	13	NA	201	NA	3	NA	1,642	NA	21	NA	674	NA	9
7	Metal ores	54	83	555	1,038	30	86	907	1,505	49	125	338	855	18	71
8	Other mining	387	668	83	86	32	57	16	17	6	12	8	10	3	6
9	Mining support	9	78	183	447	2	35	91	188	1	15	40	117	0	9
10	Food	2,286	4,040	317	359	724	1,448	141	123	323	498	50	38	114	153
11	Beverages	279	423	354	394	99	167	128	187	36	79	57	76	16	32
12	Tobacco	27	30	821	564	22	17	1,707	486	46	15	812	223	22	7
13	Textiles	882	1,392	265	172	234	239	43	29	37	40	17	10	15	15
14	Wearing	1,726	4,300	129	100	222	429	14	8	25	36	7	4	11	17
15	Leather	988	1,760	232	187	229	330	39	18	38	32	16	8	15	14
16	Wood Manufacturing	914	1,081	117	106	107	115	13	17	12	19	6	8	6	8
17	Paper	577	820	211	198	122	162	78	88	45	72	34	36	20	30
18	Printing	179	436	109	99	19	43	37	19	7	8	22	9	4	4
19	Refined petroleum	167	206	646	955	108	197	753	1,536	126	316	333	731	56	151
20	Chemicals	824	1,386	221	190	182	264	206	167	170	232	76	45	63	63
21	Pharmaceutical	218	255	258	378	56	96	131	182	29	46	75	81	16	21
22	Rubber and plastic	1,258	2,563	150	135	189	347	46	34	57	88	21	12	27	31
23	Other non-metallic	1,368	2,822	132	111	180	313	45	29	62	80	18	11	25	32
24	Basic metals	438	736	369	313	162	230	278	234	122	172	107	69	47	50
25	Fabricated metal	1,334	3,167	131	113	175	359	28	24	37	76	14	10	18	30
26	Computer and electronic	465	678	243	275	113	187	122	138	57	94	49	40	23	27
27	Electrical equipment	551	934	254	258	140	241	90	81	50	75	39	29	21	27
28	Machinery	1,384	2,282	158	160	219	365	52	54	73	123	25	19	34	43
29	Motor vehicles	767	1,093	386	462	296	505	206	290	158	317	73	85	56	93
30	Other transport	135	257	216	465	29	119	54	172	7	44	20	61	3	16
31	Furniture	1,015	1,646	105	112	107	184	14	16	14	26	5	7	6	11
32	Other manufacturing	452	927	133	107	60	100	21	19	10	17	11	9	5	8
33	Repair of machinery	54	786	192	135	10	106	49	19	3	15	28	11	2	8
	Total Manufacturing	18,750	34,874	207	194	3,872	6,748	85	77	1,599	2,695	34	28	646	987

Source: Our elaboration. Monetary values are presented in BRL 1M (millions of reais) for firm averages, and in BRL 1B (billions of reais) for the total sectoral values. Number of workers are in units for firm averages and in thousands of workers for the industry total.

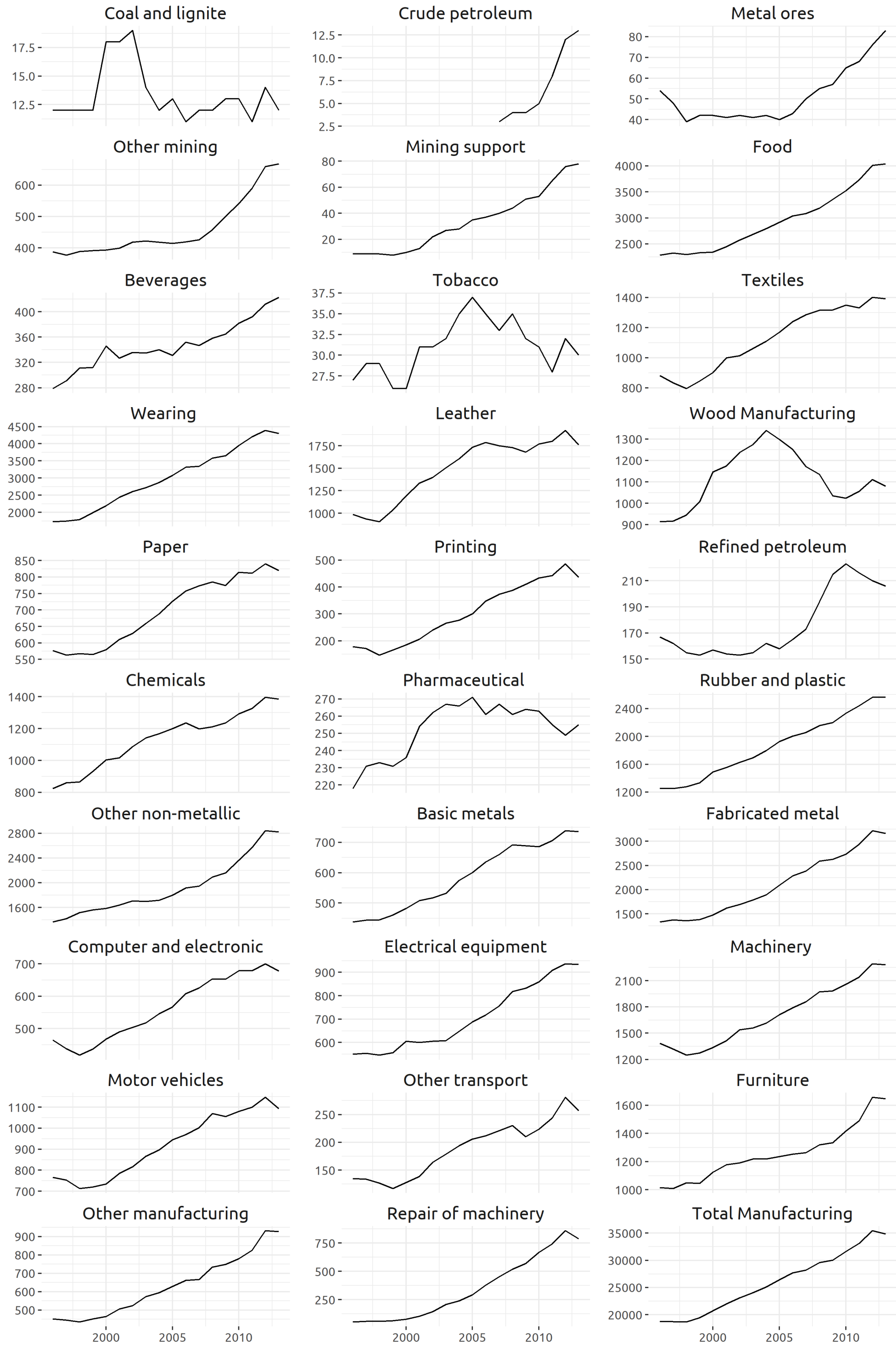


Figure 4 – Timeline of number of firms in Manufacturing.

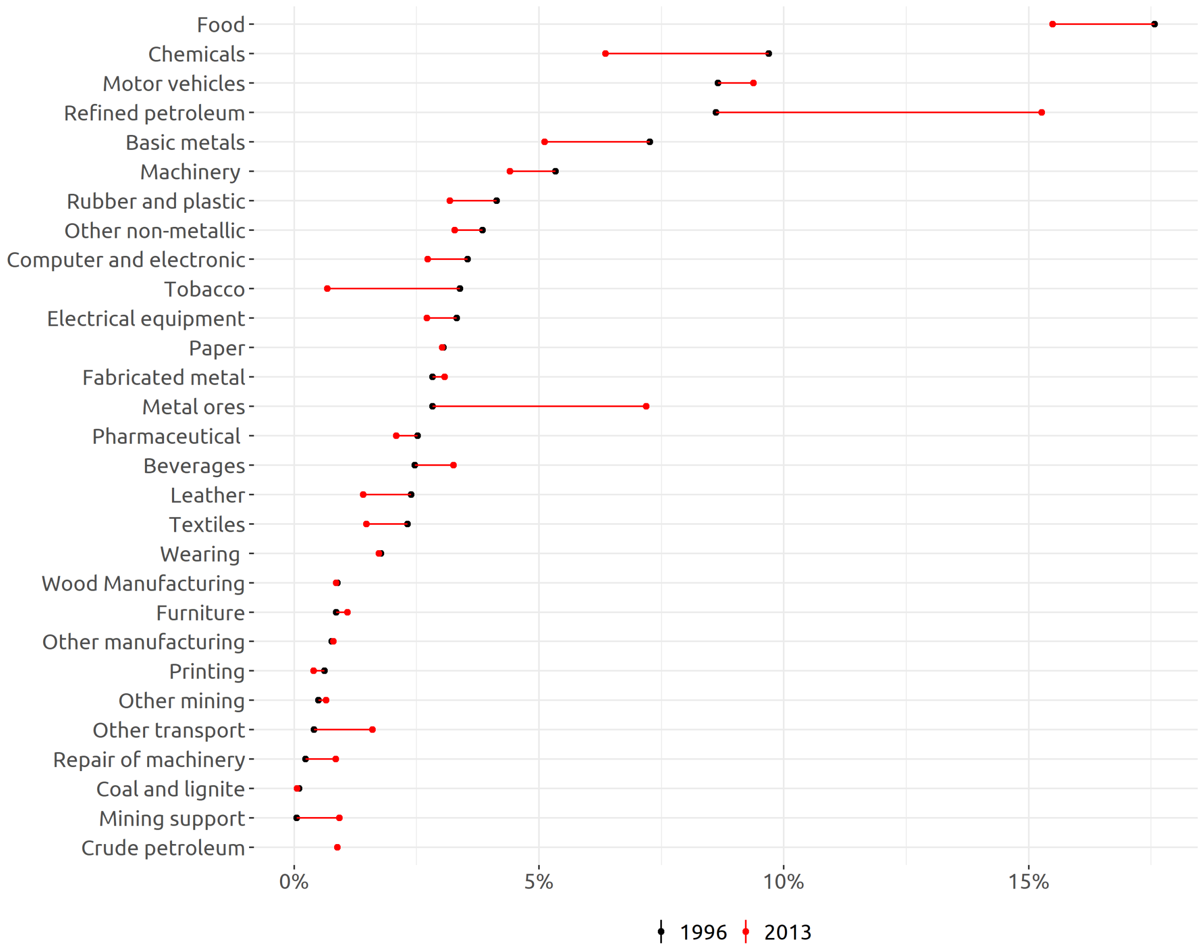


Figure 5 – Decreasingly ranked shares in value added for each sector - 1996 and 2013.

Figure 5 shows the changes in shares of value added in the period of 1996-2013. Most sectors reduced their participation to refined petroleum and metal ores. In the same period, these two industries, together with soybeans, produced the main products exported from Brazil. This decline in the complexity of manufactured and exported goods from Brazil has been appointed as a cause of low economic dynamism (HAUSMANN; HIDALGO, 2014), which is certainly observed due to the poor overall growth experienced in the period, and as a possible symptom of Dutch disease (GALA *et al.*, 2017). In fact, several studies already pointed to the failure of economic policies to improve the capacities of Brazilian industry (NEGRI; CAVALCANTE, 2014), and the low technological intensity demonstrated in most sectors (NEGRI; CAVALCANTE, 2015) is a cause of concern due to their consequence in wealth concentration and increased gap of income against developed countries (HARTMANN *et al.*, 2017).

2.2 Methodology

This work presents three exercises: basic statistics, concentration indexes and empirical density distributions estimations with their parametric fitting. The first two exercises will permit us to give a broad categorization of Brazilian Manufacturing and the heterogeneity of its performance metrics, while the last exercise permits us to assess the evidence of more delimited stylized facts, specifically, the lognormal and Pareto shape of firm size distributions and the Laplacian shape of firm productivity, growth and productivity variation distributions.

The analysis is performed in three contexts: a) an annual context, where all the data sample from each year, regardless of the sector, is pooled; b) a sectoral context, where data from all years is pooled by sector and finally c) a cross-sectional context, where all data is pooled.

Due to space limitations, the visualization of the annual context is limited to three periods (1996, 2004 and 2013) and, in the sectoral context, to five 2-digit sectors - mining of metal ores (07), manufacture of food products (10), manufacture of wearing apparel (14), manufacture of chemicals and chemical products (20) and manufacture of motor vehicles, trailers and semi-trailers (29). These periods and sectors were deemed as the most representative of the sample, considering differences in technological intensity, number of firms and share of value added in Total Manufacturing.

Concentration Indexes

In this exercise, we will use one market index to access the tail concentration of each sector[5]. The C_4C_{20} is the descending sum of market-shares of the four largest firms over the market-share of the 20 ones:

$$C_4C_{20} = \frac{\sum_{i=1}^{4} s_i}{\sum_{i=1}^{20} s_i} \tag{2.1}$$

where s_i represents the market-share of the i-th firm measured in percentages of total revenue, value added or number of employees. Any sector with less than 20 observations is excluded.

Probability Density Distributions

This exercise will explore the empirical distributions of the most important proxies of performance, size and growth for Brazilian Manufacturing. This information will help us verify, at least visually, the quality of adjustment of different classes of parametric distributions against the data. They will be vital, thus, to verify if any of the stylized facts explored in the first chapter applies to the Brazilian case.

[5] We also calculated the classic Herfindahl-Hirschman index. These results are still under analysis.

This section draws largely on Silverman (1986), Tsybakov (2009) and Scott (2015). Density estimations are smoothed versions of histograms that don't suffer from the origin point bias, which can dramatically alter the format of the underlying distribution. The histogram shows bins either with the number of observations or the relative share of occurrences between two points, characterizing a discrete visualization of the problem.

Density distributions, on the other hand, provide a curve describing the distribution of the whole sample, and the area under this curve provides the probability of ocurrence of an event - thus characterizing a continuous description of the problem (SCOTT, 2015). As a histogram can be defined by its starting point and by a bin width, the density distribution is defined by its kernel and bandwidth. The kernel function determines a curve that weights the contribution of each observation given their distance from a central point and the bandwidth determines the distance between two central points. For a uniform kernel, e.g., all the observations between two central points have the same contribution, regardless of their distance. In a Gaussian kernel, otherwise, they have decaying contributions based on how far they are from the central point.

The formal definition of the kernel function is:

$$\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right) \tag{2.2}$$

where h represents the bandwidth or smoothing parameter, K represents the desired kernel function and n represents the number of observations, x represents each central point, and x_i refers to the i-th observation in the sample. Given that the estimation of density distributions by using Equation (2.2) is computationally expensive, in general, Scott (2015) suggests the use of Discret Fourier Transforms. Those algorithms are based in data binning, where each kernel is weighted by its respective observed absolute frequency[6].

In this work, we are going to use the Epanechnikov Kernel, which is defined as below:

$$K(u) = \begin{cases} \frac{3}{4}(1 - u^2), & \text{if } |u| \leq 1 \\ 0, & \text{otherwise} \end{cases} \tag{2.3}$$

where u represents the distance between x and x_i. The Epanechnikov kernel is a 2-order kernel which is optimal in the sense that it minimizes the asymptotic mean integrated squared error (AMISE). Higher order's kernels offer the advantage of asymptotically higher precision, but they also can lead to locally negative values, which justifies our choice.

The bandwidth is selected according with Silverman's rule of thumb (SILVERMAN, 1986, pp. 48):

$$h = 0.9 \min(\sigma, \frac{\text{interquartile range}}{1.34})\, n^{-1/5} \tag{2.4}$$

[6] For a detailed description of the advantages and precision of this method see Fan and Marron (1994), Wand (1994), Hall and Wand (1996), Holmstrom (2000) and Sain (2002).

where σ represents the standard error and the interquartile range is the difference between the 75th and the 25th percentiles. In our study, we estimated the distributions for any given year or sector that have at least 300 observations. We also established 512 equally spaced bins for each distribution as a standard[7].

In order to avoid any bias due to the potential choice of a particular starting point, Scott (2015) also suggests to set the starting point of the empirical distribution far smaller than the smallest observation, so the first central point has value zero. Then, they increase as they approximate the first observations, and start to decrease after the last observations. This eliminates the origin point bias found in histograms and provides a continuous description of the phenomena.

Each density plot will be accompanied by a normal distribution fit to serve as a benchmark. The normal fittings were made using maximum-likelihood estimation.

Pareto Distributions

This section draws largely on Bottazzi *et al.* (2015). Consider that the size distribution follows a cumulative distribution function (CDF) given by:

$$F(x) = \text{Prob}(S \leq x_i) = 1 - \left(\frac{x_{\min}}{x_i}\right)^{\frac{1}{\gamma}} \tag{2.5}$$

where $\text{Prob}(S \leq x)$ represents the probability of a random value sampled from the distribution to be smaller than x_i, γ represents the format of the tail, S is a random variable, and $x_{\min}$ represents the smallest observation considered (i.e., the cut-off point from which the right tail of the distribution is modeled). Also, notice from the previous chapter that:

$$\gamma = \frac{1}{\alpha} \tag{2.6}$$

Then, making the x_i decreasingly ranked, the Hill estimator (HILL, 1975) is defined as:

$$\hat{\gamma} = \frac{1}{n-1}\sum_{j=1}^{n} \ln(x_j) - \frac{n}{n-1}\ln(x_{\min}) \tag{2.7}$$

where n represents the number of observations used until the cut-off point $x_{\min}$. Equation (2.7) includes a correction for small sample bias and constitutes a Maximum Likelihood

7 Strange as this may sound, the number of bins is not limited by the number of observations, since we are just establishing points of measurement for our empirical distribution. This is clear with a Gaussian kernel, where all observations are weighted for every central point, regardless of how far they are. What in fact limits the number of bins is the trade-off between bias and variance. Increasing the number of bins oversmooths the underlining distribution, decreasing the bias, but at the same time increasing the variance. Reducing their number has the opposite effect. But as the minimum and maximum points are not limited by the maximum and minimum value of the observations, the computational implementation just use some method to determine the optimal bandwidth - such as “solving the equation”, Silverman’s Rule of Thumb or Scott’s Rule - and set the minimum and maximum points of “measurement” accordingly, in order to guarantee the required number of bins. The number of bins are usually implemented in powers of 2 to reduce computational costs, as a fast Fourier transformer (FFT) algorithm is used.

estimator, being asymptotically Normal and efficient with smooth distributions [8]. So, it holds that:

$$E[\hat{\gamma}] = \gamma \quad \text{and} \quad Var[\hat{\gamma}] = \frac{1}{n-1}\gamma^2 \tag{2.8}$$

Since the Hill Estimator can be a poor estimator when the true distribution of the CDF is not linear[9] and since the point estimation is very dependent of the cut-off point chosen in the tail, we also did a log-rank regression using the whole distribution and a binned equipopulated empirical distribution for each plot.

It is possible to write a complementary cumulative distribution function (CCDF) of Equation (2.5) as:

$$\begin{aligned} R(x) &= \text{Prob}(S > x) = 1 - \text{Prob}(S \leq x_i) \\ &= 1 - 1 + \left(\frac{x_{\min}}{x_i}\right)^{\frac{1}{\gamma}} \\ R(x) &= \left(\frac{x_{\min}}{x_i}\right)^{\alpha} \end{aligned} \tag{2.9}$$

While Equation (2.5) gives the probability of some random value being smaller than x_i, Equation (2.9) gives the probability of some random value being greater than x_i. The parameter α is decreasing because the more extreme the value chosen for x_i the smaller the probability to find any value higher. Particularly, when $\alpha = -1$ Equation (2.9) is reduced to the so called Zipf Law, a discrete distribution used to describe various physical and social phenomena, as reviewed in Chapter 1. The $R(x)$ distribution can be estimated for a sample by:

$$\hat{R}(x) = \frac{j}{n} \tag{2.10}$$

where j represents the rank of the firm decreasingly ordered and n represents the sample size. Equation (2.10) is an empirical survival function, or alternatively, a discrete complementary cumulative distribution function. By taking the log-transformation on both sides we have:

$$\log(\hat{R}(x)) = \hat{\alpha}\log(x_{\min}) - \hat{\alpha}\log(x_j) \tag{2.11}$$

with $\alpha\log(x_{\min})$ being the scale factor for the probability function to sum up to unity. In practice, we can use the ranking j directly, since the number of observations doesn't affect the value of α, as it is a constant:

$$\log(j) = \alpha\log(x_{\min}) + \log(n) - \alpha log(x) \tag{2.12}$$

[8] See (BOTTAZZI *et al.*, 2015, footnote 6) for a discussion and list of references.

[9] Basically because of misspecification bias due to an incorrect functional form.

The parameters in this equation can then be estimated by a simple OLS regression. This procedure is called a OLS-Rank regression.

Both OLS-Rank and Hill deliver close point estimations for the true value of the coefficient α for the same $x_{\min}$ (BOTTAZZI *et al.*, 2015) when $X \geq x_{\min}$ follows a power law, but the Hill Estimator is preferred by its properties.

The exercises were conducted for the different contexts expressed at the beggining of this section. At this time, we were unable to use maximum likelihood methods to establish the optimal cut-off point of the Hill estimator. So, we set the cut-off point on the 500th observation (DOSI *et al.*, 2008). For the OLS-Rank regression, we used all the data in the respective context. Also, following Newman (2005) and Clauset *et al.* (2009), instead of reporting the value of $|\alpha|$ for the CCDF, we report the values of $|\alpha| + 1$, as this gives the decay value of the PDF distribution.

Validation tests on the quality of the adjustment of specific distributions against the data will be done in the future, following Clauset *et al.* (2009).

Subbotin Fit

For parametric estimations of productivity, growth rates and productivity change we use the Asymmetrical Exponential Power densities (AEP), a class of distributions introduced by Bottazzi and Secchi (2011) which belongs to the Subbotin Family of parametric fits (SUBBOTIN, 1923). This distribution is composed by five parameters, which present both Laplacian and Gaussian distributions as special cases. Its functional form is:

$$f_{\mathrm{AEP}}(x; b_l, b_r, a_l, a_r, m) = \frac{1}{C} e^{\left(-\left[\frac{1}{b_l}\left|\frac{x-m}{a_l}\right|^{b_l}\theta(m-x) + \frac{1}{b_r}\left|\frac{x-m}{a_r}\right|^{b_r}\theta(x-m)\right)\right]} \tag{2.13}$$

with

$$C = \frac{a_l}{b_l}^{\frac{1}{bl}-1}\Gamma(1/b_l) + \frac{a_r}{b_r}^{\frac{1}{br}-1}\Gamma(1/b_r) \tag{2.14}$$

where $\theta(x)$ and $\Gamma(x)$ are, respectively, the Heaviside theta and the Gamma function, x represents the sample of the variable for which we want to estimate the parametric fit, m is the sample average, a_l and a_r are the left and right scale parameters, respectively, and b_l and b_r are the shape parameters.

Specifically, when $b = 1$ the fit identifies a Laplacian distribution, and when $b = 2$ it becomes a normal distribution. The AEP allows each tail to be determined independently, and the lower the b, the fatter the tail. The parameters are estimated using maximum likelihood estimation, following Bottazzi and Secchi (2011).

2.3 Results

Concentration Indexes

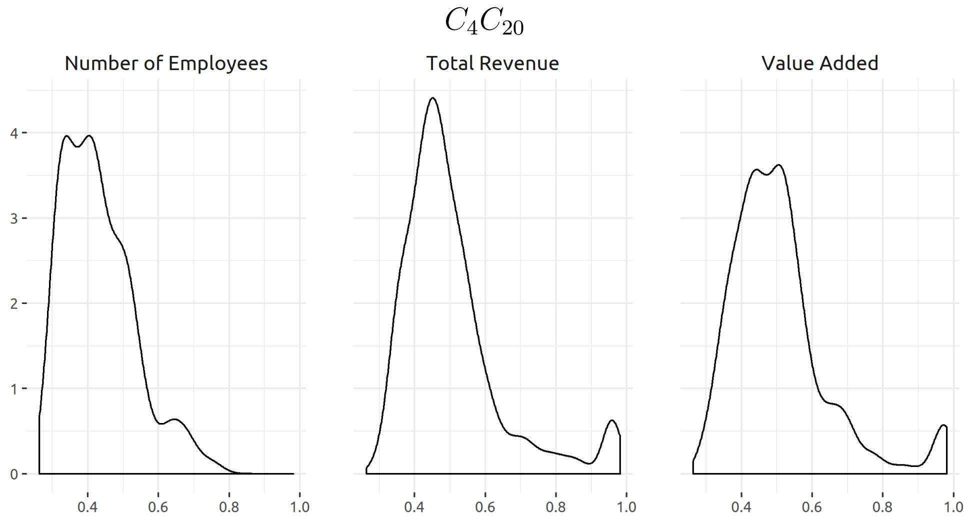
In this section we present the results of concentration indexes. In order to summarize the information, Figure 6 presents a density estimation of the indexes estimated for all periods, sectors and proxies for firm size. The shape of the distributions is very similar, with modes around 40% and 50%. This means that, on average, in most sectors the 4 top firms have as much employees, revenue and value added as the other 16 from the top 20 firms. Therefore, tails are heavy.

The transition from the second level of disaggregation to the third level changes the format of the distributions, notably for value-added. Although the modes remain somewhat stable, the dispersion increases significantly. As a consequence, concentration indexes appear very sensitive to the disaggregation level[10].

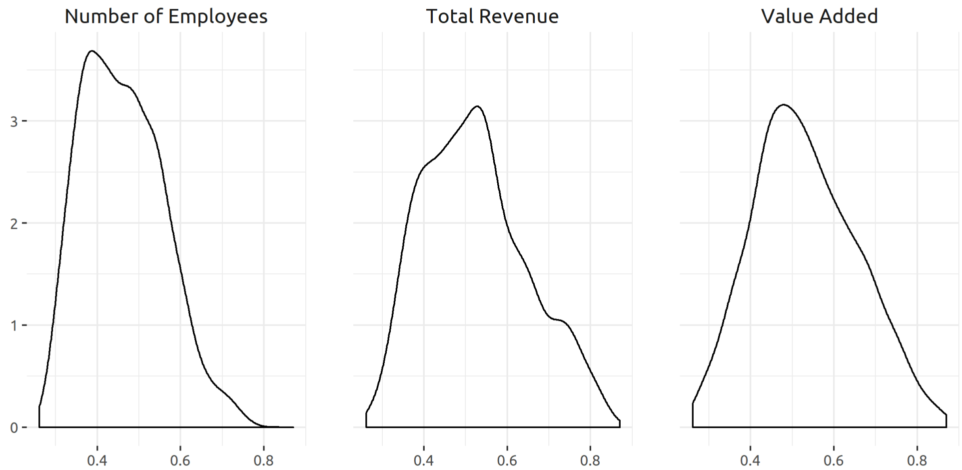
Next we investigate if these patterns are persistent over time. Figure 7 presents annual Tukey-style box plots of the index C_4C_{20} of all sectors by year. Estimated medians are close, showing little variation for the whole period, independently of the proxy used for firm size. The deviations are different though, and as in the density probability plots, increase in the more disaggregated view.

What we can conclude from this is that, despite the high concentration in the tails, independently of the proxy, sector or period, concentration indexes *per se* can be very agreeable when estimated considering a specific level of disaggregation, but show contrasting results for other levels. Since firms are not product-specific, i.e., they are a collection of different brands, products and services that meet different necessities of customers with different profiles, the attempt of finer grain sectoral classifications towards specific products seems a failed battle.

[10] Although we don't present it, we found the same sensitivity using the Herfindahl-Hirschman index.



(a)



(b)

Figure 6 – Probability Density Plots for annual-sectoral concentration indexes. Concentration is measured using number of employees, total revenue and value added as proxies for firm size (a) at 2-digit ISIC level and (b) at 3-digit ISIC level.

Notably, the trend of integration between products and services, especially in the IT sector, made some lines between Manufacturing and other sectors more blurred. Does Apple is a service or a manufacturing company? There is no clear answer. Industrial surveys of course try to separate the manufacturing gains from the services, but this cut is relatively arbitrary. So, what we are trying to point is that concentration metrics can be highly misleading for antitrust policies and analyses, and should be used with caution.

This also points to the challenge of measuring market selection and competition with this broad definition of sector. It is to be expected that not all firms in a given sector produce for the final consumer, and certainly they don't compete in the same market niches. While we tentatively recreated some of the results from the literature in the next chapter regarding the nature of productivity change, it should be noted that any study

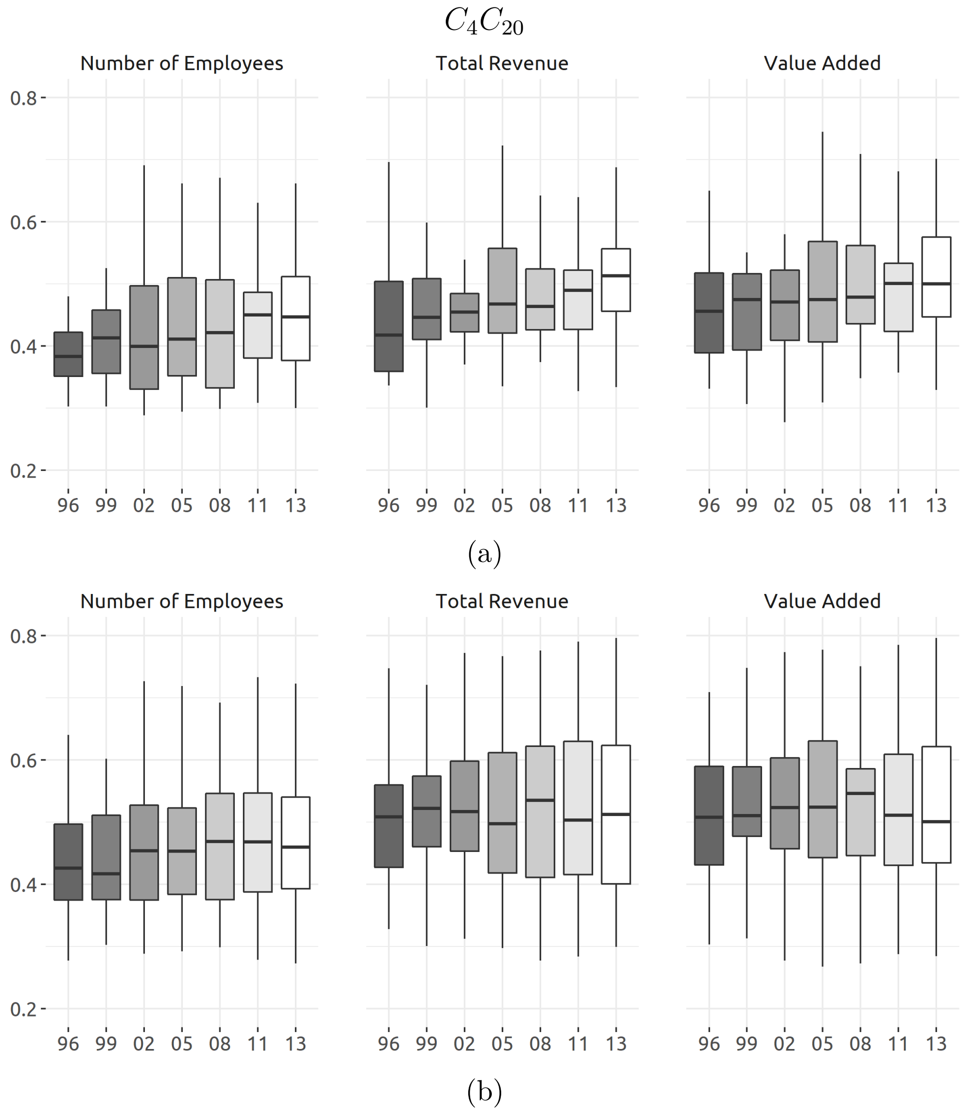


Figure 7 – Tukey Box-plots for annual-sectoral indexes. The central line represents the median. Lower and upper hinges show the first and third quartiles. Whiskers show the last observation within at most 1.5 times the interquatile range from the hinges. Concentration is measured using number of employees, total revenue and value added as proxies for firm size (a) at 2-digit ISIC level and (b) at 3-digit ISIC level.

that does not analyze firm competition at the product level (and even so, with specific clusters referring to different qualities of goods) will produce only very rough answers (DOSI *et al.*, 2015).

Also, the regional nature of competition, while ameliorated by the creation of online commerce, can produce local monopolies that would be transparent in this kind of analysis. Does the top 20 firms are in fact competing in the same regions for the same consumers or some of them are regional leaders? In a continental country such as Brazil it is not realistic to expect that all firms are in the same markets fighting for the same consumers. Questions of this kind are challenging, and we are only at the beginning of providing answers, which will require a much broader integration of different datasets. Among them, microdata of prices, fine grain clusterization of products in meaningful niches and spatial delimitation of competing markets.

Overall, the high concentration of the markets, which share a common mode across the different metrics, points towards the existence of a particular structure regulating the market functioning, which we will explore in more detail in the next section.

Size Distributions

The objective of this section is to search for characteristic patterns in the market structure using different proxies of size. The existence of a particular shape, in light of what was reviewed in the previous chapter, may suggest a particular mechanism behind market organization. When this commonality is shared across countries it creates favorable signs for the existence of processes that are of a pure economic nature, trespassing cultural and regional differences.

We begin by displaying basic statistics for number of employees, total revenue and value added. These are the most common metrics for firm size and are what we commonly define as the "market" from the supply side.

Table 3 shows the moments for each of these proxies in the cross-sectional view of our sample. Most sectors have indeed significant positive skewness and kurtosis, which for unimodal distributions means that they are fatter on the left side with long tails on the right side. We know that this dispersion comes mostly from large enterprises, and this becomes apparent by the distance between average and median in most sectors. Particularly in Metal ores (ISIC 7), the average is almost 8 times the median for number of workers, 34 times for total revenue and 27 times for value added. That is a significant result, causing the fat right tail.

The conclusions are similar for Total Manufacturing. The standard deviation is much larger than both the average and the median, showing the importance of right-hand side extreme values to determine the format of the distribution. In fact, the distributions of these proxies are so extreme that it is not possible to have a meaningful visualization of their shapes as they are. Therefore, our plots will present the values from either the log of the variable or the log-rank version of the complementary cumulative distribution function (CCDF).

Figure 8 and 9 shows the estimated probability density distributions for the natural logarithm of size proxies. The dotted line in each plot represents the fit of a normal distribution[11] using maximum likelihood estimations. Results are depicted for Total Manufacturing in three years of our sample. The shapes present at first glance a shift to the left in all metrics.

[11] That is, the original distribution is fitted by a lognormal fit.

Table 3 – Firm Size in Brazilian Manufacturing - Cross-Sectional data from 1996 to 2013

ISIC	Industry	Total Obs.	Number of Workers					Total Revenue					Value Added				
			Avg.	Median	Sd.	Skew.	Kurt.	Avg.	Median	Sd.	Skew.	Kurt.	Avg.	Median	Sd.	Skew.	Kurt.
5	Coal and lignite	240	354	326	280	1	2	87	67	75	1	2	44	35	42	1	4
6	Crude petroleum	54	172	99	173	3	10	1,052	287	1,690	2	9	417	122	619	2	5
7	Metal ores	928	827	128	4,306	10	104	1,850	54	11,193	9	99	808	29	4,668	10	104
8	Other mining	8,269	82	53	95	5	36	15	7	35	9	114	8	4	21	11	170
9	Mining support	614	436	236	502	2	7	184	72	280	3	12	117	49	187	3	15
10	Food	52,966	337	74	1,535	23	837	136	15	811	19	465	42	4	263	19	467
11	Beverages	6,239	312	83	1,074	12	196	173	12	1,065	17	332	74	4	515	16	292
12	Tobacco	559	609	138	1,343	4	22	760	51	2,526	7	59	333	18	1,318	7	57
13	Textiles	20,244	203	78	557	15	303	35	8	107	12	207	13	3	45	14	294
14	Wearing	53,903	102	54	331	35	1,893	9	2	42	18	453	4	1	21	20	560
15	Leather	26,843	195	68	847	17	387	24	3	117	14	263	10	2	53	18	432
16	Wood Manufacturing	20,122	109	57	215	19	668	15	3	69	27	1,113	6	2	35	37	1,930
17	Paper	12,544	191	75	442	9	129	85	10	408	11	139	36	3	186	11	140
18	Printing	5,458	101	51	219	11	151	25	5	89	10	133	14	3	51	10	135
19	Refined petroleum	3,178	735	217	3,539	13	188	1,157	76	13,650	15	235	617	24	7,780	14	216
20	Chemicals	20,382	184	75	434	9	128	184	24	787	14	324	53	8	226	15	340
21	Pharmaceutical	4,544	300	118	454	3	19	142	24	319	4	22	70	12	156	4	24
22	Rubber and plastic	33,526	133	66	297	13	263	40	10	191	19	460	15	3	74	18	419
23	Other non-metallic	34,731	115	55	265	12	221	34	3	194	20	650	15	1	86	17	454
24	Basic metals	10,546	310	85	981	8	93	264	20	1,234	9	105	92	6	465	10	115
25	Fabricated metal	37,951	116	59	233	13	315	25	6	101	18	535	10	3	37	14	292
26	Computer and electronic	10,128	242	83	533	7	76	125	14	529	11	194	38	6	163	14	348
27	Electrical equipment	12,729	235	76	822	14	271	88	12	396	12	184	31	5	149	13	218
28	Machinery	30,780	154	68	372	9	107	55	12	233	15	309	21	6	79	12	200
29	Motor vehicles	16,512	402	87	1,518	11	152	249	14	1,658	13	204	77	6	484	15	264
30	Other transport	3,402	333	85	1,191	10	118	131	9	826	11	145	46	4	277	12	173
31	Furniture	22,672	107	59	156	6	62	15	4	38	8	95	6	2	14	7	83
32	Other manufacturing	11,356	116	59	205	7	80	19	4	55	8	80	9	2	26	8	78
33	Repair of machinery	6,275	147	60	402	11	187	26	5	173	24	789	12	3	45	9	99
	Total Manufacturing	467,695	189	65	819	33	1,942	81	7	1,350	118	18,001	30	3	702	142	23,106

Source: Our elaboration. Monetary values are presented in BRL 1M (millions of reais). Number of workers are in units of headcount.

Total revenue shows evidence of the emergence of a bimodality, which we suspect to be caused by the introduction of a new tax regime privileging smaller firms[12]. This is the first time that we, as authors, see a so clear-cut effect of policy in the market structure, a fact that we pretend to investigate further.

These patterns, despite the evidence of bimodality, seem to follow rather closely the parametric distributions, with the worst case occurring when using number of employees. When we move to a sectoral view (Figure 9), the apparent quality of fit of these distributions seems to be improved, particularly for the monetary proxies, a result that is in contrast with the literature. The European and US results tended to show that the apparent lognormal shape occurred as a consequence of sheer aggregation, exposed for example in Hymer and Pashigian (1962) for UK and Bottazzi and Secchi (2003) for US. While demonstrating the same fact for the Italian industry, this was the main argument of Bottazzi *et al.* (2007) to reduce the importance of the lognormal shape as a stylized fact, instead giving emphasis to a very skewed shape.

It is important to highlight that age was showed to have an important role in these distributions. Cabral and Mata (2003) demonstrate that the distributions became less skewed when only older firms are considered. So, there is compelling evidence pointing that entry-exit dynamic is responsible for the highly asymmetrical shape found in size distributions. Unfortunately, no truly age-related data[13] of the sample used is available at this time, so it is not possible to implement the more recent advances regarding Gibrat Law's tests and age-splitted Density Probabilities of firm size distributions.

[12] The "simples", a special tax regime that was implemented in the Complementary Law number 123, from December 14th, 2006, introduces the option for firms under a certain constraint of revenue to be taxed by a fixed percentage of their sales. The limits, around R$ 2-4 million for the period, and the date of the law both coincides with the appearance of the bimodality.

[13] Some studies try to control this dynamic by setting a year as basis and classifying all firms that don't appear in that year as new. So, they create cohorts of samples of new firms and see their evolution through time. The problem with this approach is that we don't really know if the firms appearing in the dataset are really "new". In our case, e.g., since we have only census information for firms with at least 30 employees, firms that are "around" this threshold may enter and exit the survey, composing the sample one year and disappearing the next one, due to hiring practices common to economic cycles.

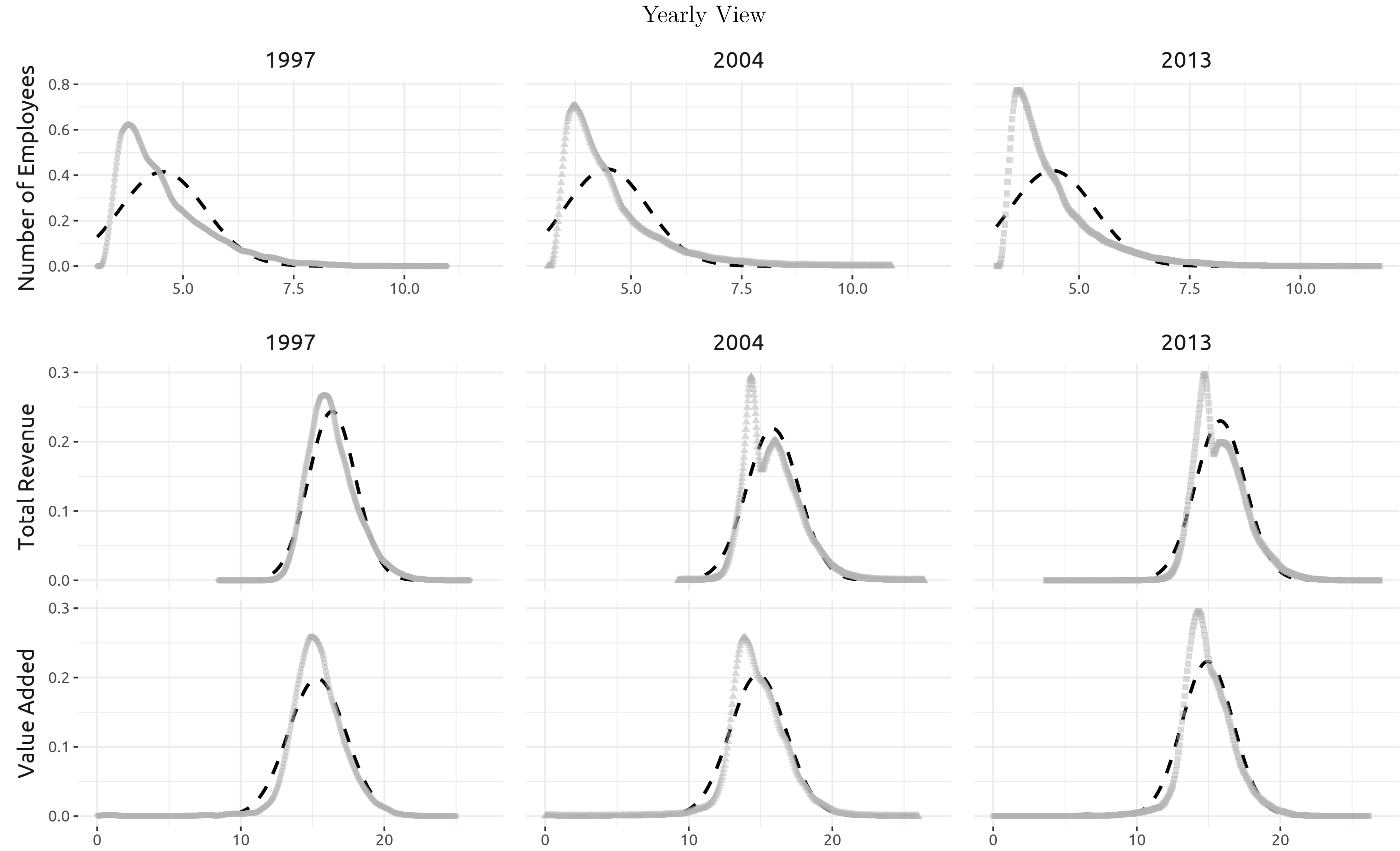


Figure 8 – Size - Annual Probability Density Plots. Variables in log, axes in level. Dashed lines represent a normal fit for each distribution.

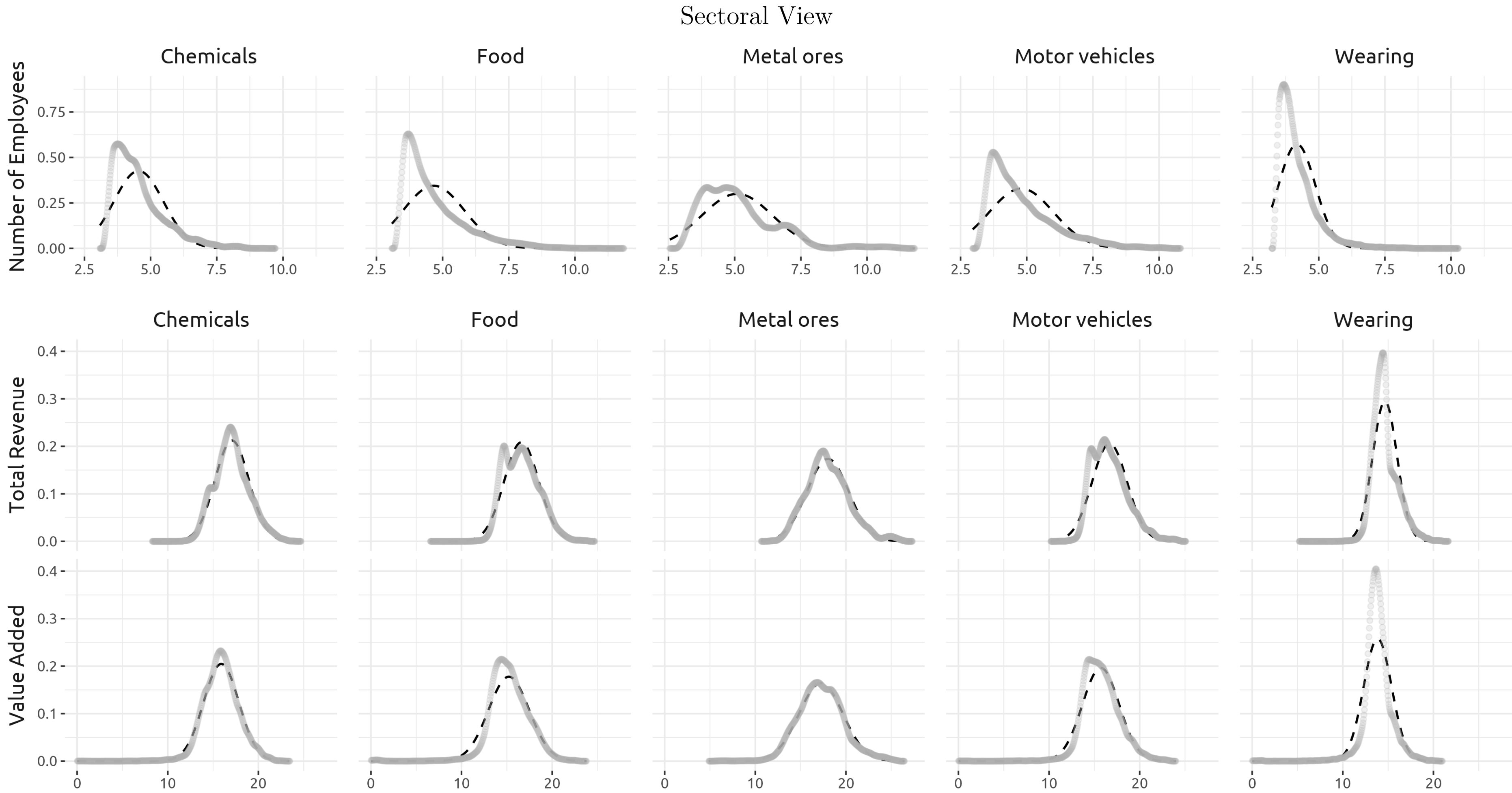


Figure 9 – Size - Sectoral Probability Density Plots. Variables in log, axes in level. Dashed lines represent a normal fit for each distribution.

Figure 10 and 11 show the distribution of these metrics in a log-rank plot[14]. In this plot we compare both the lognormal (blue) and Pareto (red) fits for each distribution. These plots shows the right tail of the distribution on the top-left side, with the body and left tail of the distribution concentrated in the bottom-right of each graph. The bimodality of Total Revenue is not visible anymore.

Both fits seem very close for the data in the annual view. Especially when the number of employees is considered, the Pareto fit seems favored over the lognormal. Monetary values, on the other hand, display a more lognormal appearence, particularly in the body. However, it is important to evaluate the robustness to disaggregation of these fits.

It is interesting to see that, generally, the quality of the adjustment of the sectoral values, as seen in Figure 11, seems even better than in the aggregate case, with value added presenting an almost perfect fit of a lognormal distribution. As in the case with the annual view, the Pareto fit seems favored only for number of employees. Different sectors and years also share similar inclinations.

In order to formally present this results, we proceed to report OLS-Rank and Hill estimations of the right tail of size distributions. For the Hill Estimator we considered the five hundred biggest firms in each context, whereas for the OLS-Rank we used the whole distribution. The estimations are presented in Table 4. The OLS-Rank showed great explanatory power of the model, in general over 90%, which we don't report in detail here. This result should be understood as the model being generally a good "fit" for the data rather than suggesting the superiority of any particular distribution.

Despite that, the high explanatory power is similar to what was found by Axtell (2001) for US manufacturing, which is particularly surprising since our method is much more precise, then less condescending with deviations[15].

More interestingly, sectors that present non-smooth formats or bimodalities are still very well represented by the model. Of all proxies, value added is the one with the "poorest" fit, which, as shown visually in the previous graphs, reflects the apparent superiority of the lognormal fit. A similar result was found by Dosi *et al.* (2008), regarding the evidence of a concavity. Yet, more investigation is still necessary to compare the quality of fit of different parametric distributions with the use of formal tests (CLAUSET *et al.*, 2009).

The OLS-Rank coefficients vary from 1.62 to 2.33 for number of workers, 2.11 to 3.28 for total revenue and 2.24 to 3.28 for value added. The same metrics using the ML Hill estimator provide coefficients that range from 1.60 to 3.07, 1.47 to 2.89, and 1.33 to 2.86, respectively.

[14] In this visualization, we plot the previous adaptation on the complementary cumulative distribution function, i.e., we took the logarithm of the decreasingly ranked firms and plotted it against the logarithm of the proxy used to measure size.

[15] Axtell used a binned probability function to estimate his model, which, accordingly to Bottazzi *et al.* (2015) and Clauset *et al.* (2009), is a source of significant bias.

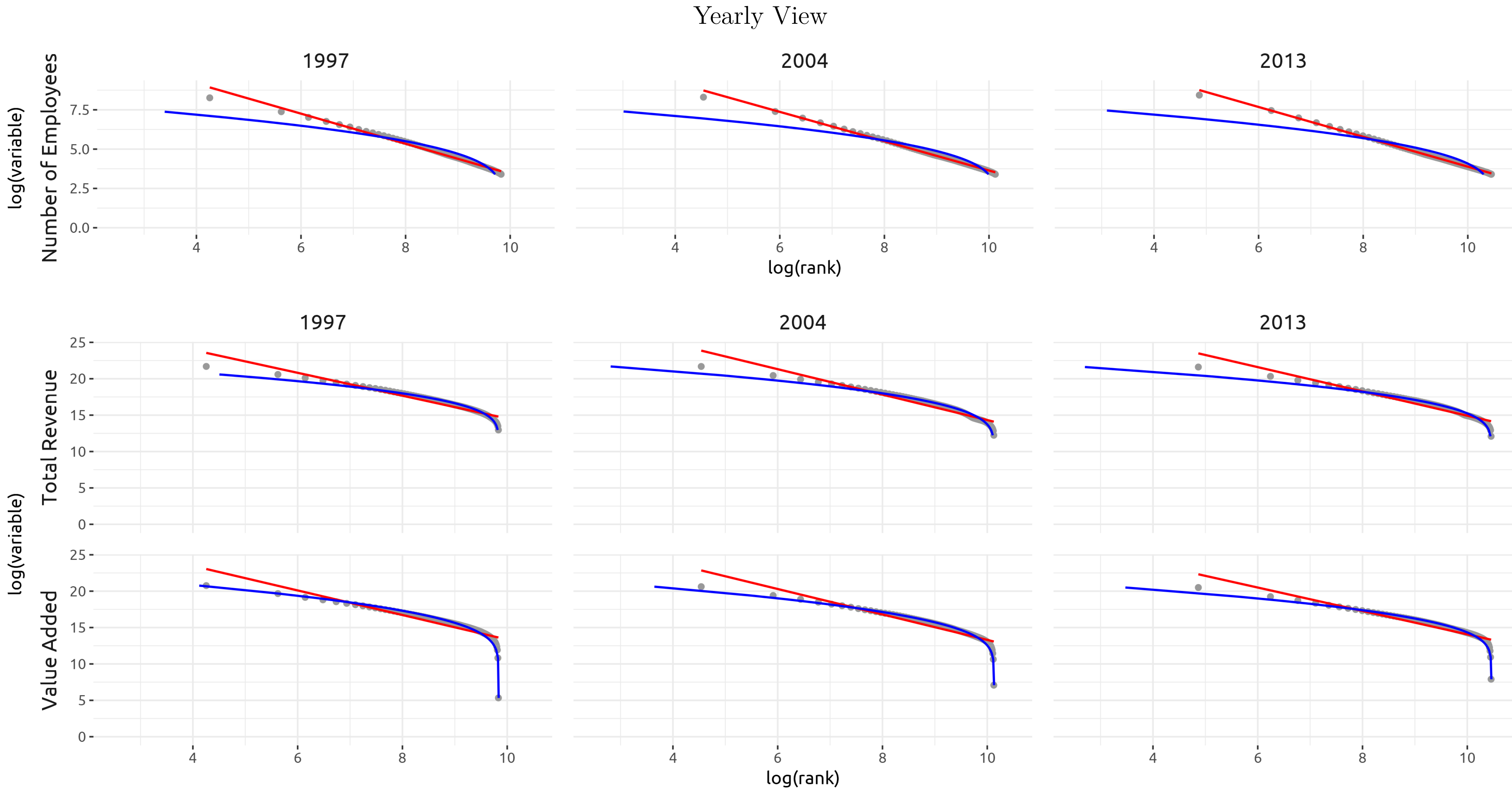


Figure 10 – Size - Annual Log-Rank plots. The red line represents the Pareto fit from the OLS-Rank estimation, while the blue line represents the lognormal fit using maximum likelihood estimation.

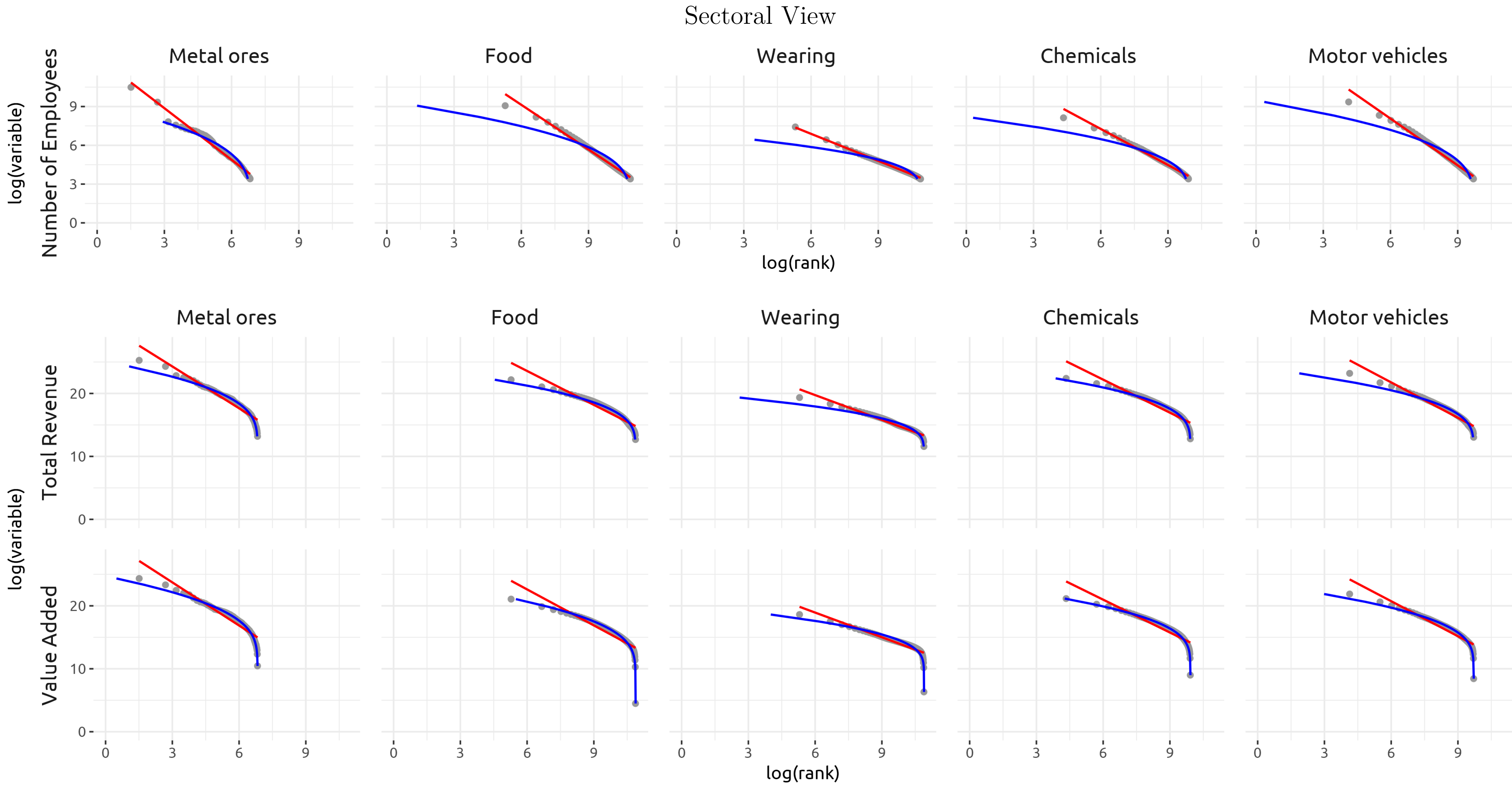


Figure 11 – Size - Sectoral Log-Rank plots. The red line represents the Pareto fit from the OLS-Rank estimation, while the blue line represents the lognormal fit using maximum likelihood estimation.

The estimates obtained by the two methods are close for some sectors, but generally don't agree. This should come as no surprise. As they are both very precise, their differences are basically caused by differences in the cut-off value, with the Hill method being very sensitive to the choice of $x_{\min}$.

In general, our OLS-Rank estimates for Total Manufacturing are in accordance with the literature, with a coefficient of 1.94 when using total employees to measure size, whereas Axtell (2001) found a slope of 2.06 for the US. Our results for total revenue, though, are less agreeable. While Axtell (2001) found a slope of 1.99 and Dosi *et al.* (2008), using Italian and French firms, a range between 1.8 and 2.05, our results show coefficients for the OLS-Rank and Hill estimates equal to 2.7 and 2.45, respectively.

Size distributions, therefore, present significant right-skewed distributions regardless of the metric used, which closely resemble lognormal distributions for total revenues and value added and Pareto distributions for number of employees. This pattern seems robust to both different disaggregation levels and time frames.

The great heterogeneity evidenced by these distributions also corroborates the outcomes of other works for the Brazilian economy (NOGUEIRA *et al.*, 2014; ESTEVES, 2015; SQUEFF; NOGUEIRA, 2015; CATELA *et al.*, 2015). Why so firms in the same sector face such a dissimilar performance and set of characteristics?

A possible explanation could be that, beyond the heterogeneity of firm capabilities, gains from different scales of operation and access to better prices through suppliers, market niches and brand power would create earning differentials that would not be mitigated even with firms sharing the same costs or technology (STURGEON, 2002). Due to the widespread heterogeneity in performance metrics found for other countries (GRILICHES; REGEV, 1995; BARTELSMAN; DOMS, 2000), this heterogeneity is not necessarily a problem in itself, although the wealth concentration caused by this variability certainly is (ATKINSON; PIKETTY, 2007). In fact, heterogeneity may very well be a feature of the system, and hierarchies can constitute an easier way to transmite signaling information, helping to organize markets (KRUGMAN, 1996b). The idea that markets are intelligent and self-organized come as back as Hayek (1945), but the atomized information that the market contains is not necessarily optimal under a set with uniform agents. In fact, studies from network theory show that networks following power laws are very robust to random shocks (or, in our case, bankruptcy e.g.) since there are few large hubs and many small components, while also reducing the distance between agents (BARABASI, 2016). At the same time, they are much more fragile against target failures, or meltdowns of important players, which in economics generated the concept of "too big to fail" (NURISSO; PRESCOTT, 2017), popularized in the post-2008 crisis after the rescue of several financial and industrial firms. If size is a good proxy of the number of transactions and the number of other individuals a firm is connected, then this heterogeneity may imply some kind of power law or similarly robust network of producers, a promising venue of research.

Table 4 – Pareto Coefficients from OLS-Rank and Hill Estimations for Firm Size in Brazilian Manufacturing - Cross-Sectional Data

ISIC	Industry	Number Workers				Total Revenue				Value Added			
		OLS-Rank	Hill			OLS-Rank	Hill			OLS-Rank	Hill		
		$\alpha+1$	$\alpha+1$	95% Interval		$\alpha+1$	$\alpha+1$	95% Interval		$\alpha+1$	$\alpha+1$	95% Interval	
7	Metal ores	2.33***	1.79	1.72	1.86	3.21***	1.47	1.43	1.52	3.28***	1.48	1.44	1.53
8	Other mining	1.62***	2.73	2.58	2.89	2.11***	2.40	2.28	2.53	2.24***	2.31	2.19	2.42
9	Mining support	2.06***	1.64	1.58	1.70	2.25***	1.67	1.61	1.73	2.3***	1.57	1.52	1.62
10	Food	2.15***	2.58	2.44	2.72	2.8***	2.24	2.13	2.35	2.9***	2.17	2.07	2.28
11	Beverages	2.13***	2.05	1.96	2.15	2.91***	1.94	1.86	2.02	3.14***	1.86	1.78	1.94
12	Tobacco	2.32***	1.60	1.55	1.65	3.22***	1.30	1.28	1.33	3.24***	1.33	1.30	1.35
13	Textiles	1.97***	2.71	2.56	2.86	2.53***	2.78	2.62	2.93	2.53***	2.62	2.48	2.76
14	Wearing	1.7***	2.39	2.27	2.51	2.31***	2.52	2.38	2.65	2.31***	2.43	2.30	2.56
15	Leather	1.91***	2.18	2.08	2.29	2.52***	2.38	2.26	2.50	2.38***	2.26	2.15	2.37
16	Wood Manufacturing	1.76***	2.95	2.78	3.12	2.28***	2.21	2.10	2.31	2.36***	2.24	2.13	2.35
17	Paper	1.95***	2.57	2.43	2.71	2.61***	1.90	1.82	1.98	2.73***	1.81	1.74	1.88
18	Printing	1.73***	2.44	2.31	2.56	2.45***	2.03	1.94	2.12	2.42***	2.06	1.96	2.15
19	Refined petroleum	2.16***	2.42	2.29	2.54	2.3***	2.17	2.07	2.27	2.46***	2.14	2.04	2.24
20	Chemicals	1.93***	2.58	2.45	2.72	2.75***	2.45	2.33	2.58	2.74***	2.44	2.32	2.57
21	Pharmaceutical	2.07***	3.02	2.84	3.19	2.69***	2.49	2.36	2.62	2.72***	2.46	2.33	2.58
22	Rubber and plastic	1.81***	2.52	2.38	2.65	2.37***	2.31	2.20	2.42	2.44***	2.26	2.15	2.37
23	Other non-metallic	1.78***	2.81	2.65	2.97	2.62***	2.47	2.35	2.60	2.58***	2.43	2.31	2.56
24	Basic metals	2.1***	2.23	2.12	2.34	2.9***	1.90	1.82	1.98	2.88***	1.85	1.77	1.92
25	Fabricated metal	1.77***	2.89	2.73	3.06	2.34***	2.51	2.37	2.64	2.36***	2.59	2.45	2.72
26	Computer and electronic	2.05***	2.58	2.44	2.72	2.73***	2.01	1.92	2.10	2.67***	2.11	2.01	2.21
27	Electrical equipment	2.01***	2.32	2.21	2.44	2.59***	2.26	2.15	2.37	2.59***	2.18	2.08	2.28
28	Machinery	1.86***	2.63	2.49	2.77	2.38***	2.52	2.39	2.65	2.36***	2.48	2.35	2.61
29	Motor vehicles	2.21***	2.31	2.20	2.43	2.87***	1.95	1.86	2.03	2.85***	1.96	1.88	2.04
30	Other transport	2.13***	2.06	1.97	2.16	2.85***	1.85	1.77	1.92	2.89***	1.88	1.80	1.95
31	Furniture	1.74***	3.07	2.89	3.25	2.3***	2.89	2.72	3.06	2.42***	2.86	2.70	3.03
32	Other manufacturing	1.78***	2.69	2.54	2.84	2.36***	2.45	2.33	2.58	2.43***	2.40	2.28	2.53
33	Repair of machinery	1.87***	2.31	2.19	2.42	2.33***	1.91	1.83	1.99	2.28***	2.01	1.92	2.10
	Total Manufacturing	1.94***	2.91	2.74	3.07	2.7***	2.45	2.33	2.58	2.68***	2.37	2.25	2.49

Source: Our elaboration. Stars represent significance at the 1% level. $\alpha+1$ refers to the inclination of the PDF, as reported in Newman (2005) and Clauset *et al.* (2009), while α represents the inclination of the CDF.

Productivity Distributions

We repeat the previous exercise for productivity, which we consider to be the most important metric of fitness and performance, and acts as the main mechanism of survival in evolutionary theories, forming the "replicator dynamics" of models such as Metcalfe (1994).

We use labor productivity for several reasons. The first is that the data on firm capital is unrepresentative of the whole sample due to the large amount of missing data. Labor productivity also doesn't require any intuition about the relationship of the productive structure, nor does it requires strong hypothesis about the substitution between capital and labor[16]. Finally, it guarantees comparability between our study and those of several other scholars (DOSI *et al.*, 2012; YU *et al.*, 2015b; MATHEW, 2017).

Table 5 presents the moments of labor productivity in the cross-sectional data. Also, we present the averages observed in 1996 and 2013, for comparison purposes. The sectoral values represent the simple average considering the total revenue divided by the number of employees, while the firm averages represent the sample average. The idea is that the closer these two values are, the more homogeneous is the productive structure inside the sector, i.e., firms use relatively similar technology and are in the same technological frontier[17]. The more dissimilar, the more evidence we have of scale returns, hierarchy and technological gaps. At first glance, the results show that sectors that are mainly related to commodities, such as metal ores, basic metals and refined petroleum are the ones that have the biggest discrepancies, probably due to scale returns and monopsony power. Tobacco and chemicals also make to the same list, but probably for different reasons, like luxury exports in the case of the former, and market niches in the second.

Of all sectors, only five presented increases in the average productivity of firms and twelve in the sectoral average productivity. The more favorable view of the sectoral average points to the skewed nature of the firm size distributions, where the bigger firms tend to dominate most of the market-share, and thus, disproportionally affect the sectoral metrics of performance. The distribution is positively asymmetric and has heavy tails for all sectors. An annual analysis[18] shows more in detail the movements of productivity in the period. Suffice to say that, from 1996 up to 2004, it suffers a downfall, with a recovery that just in the brink of 2013 begins to return, albeit still far, to the levels observed in 1996. The first period covers most of the commercial opening and heavy competition caused by what was an overvalued exchange rate, which appears to have had a destructive effect on national competences. However, it is uncertain if this loss of productivity is caused only

[16] Issues related to empirical estimation of these metrics and their relationship with account identities are discussed by Felipe and McCombie (2013)

[17] Of course, if the reader believes that it is possible to substitute, in a way that is economically viable, capital and labor, he will obviously disagree with this representation. We tend to see technology imposing harsh limits in this substitution, with Leontief production functions being the rule, not the exception.

[18] For brevity, we don't present the results here.

Table 5 – Labor Productivity in Brazilian Manufacturing - 1996-2013

ISIC	Industry	Total Obs.	Cross-Section					Firm Avg.		Sect. Avg.	
			Avg.	Median	Sd.	Skew.	Kurt.	1996	2013	1996	2013
5	Coal and lignite	240	135	104	110	2	8	152	140	253	235
6	Crude petroleum	54	2,678	771	5,328	5	34	NA	5,458	NA	8,155
7	Metal ores	928	449	217	590	2	10	328	299	1,633	1,449
8	Other mining	8,269	84	59	90	5	49	79	107	197	203
9	Mining support	614	294	208	274	3	14	201	273	500	422
10	Food	52,966	86	44	158	15	622	124	70	447	343
11	Beverages	6,239	142	45	745	17	375	137	130	361	475
12	Tobacco	559	236	107	332	3	17	371	199	2,080	861
13	Textiles	20,244	51	36	58	6	75	57	50	160	167
14	Wearing	53,903	27	16	52	40	3,261	35	30	112	85
15	Leather	26,843	36	25	43	8	163	54	34	167	98
16	Wood Manufacturing	20,122	41	27	60	15	592	42	40	115	162
17	Paper	12,544	80	48	221	47	3,087	83	91	368	445
18	Printing	5,458	91	53	173	11	184	150	67	340	188
19	Refined petroleum	3,178	177	103	284	6	56	173	167	1,166	1,608
20	Chemicals	20,382	185	98	282	6	91	263	158	935	879
21	Pharmaceutical	4,544	153	104	147	2	8	189	156	508	482
22	Rubber and plastic	33,526	75	52	102	13	394	100	65	303	255
23	Other non-metallic	34,731	58	26	102	6	65	72	49	342	257
24	Basic metals	10,546	123	70	200	10	199	130	103	755	747
25	Fabricated metal	37,951	63	45	78	9	168	81	64	213	212
26	Computer and electronic	10,128	102	66	158	10	185	122	95	500	503
27	Electrical equipment	12,729	86	60	111	17	780	95	75	355	312
28	Machinery	30,780	105	77	139	28	1,922	126	94	332	337
29	Motor vehicles	16,512	90	63	124	14	396	96	85	533	626
30	Other transport	3,402	71	42	115	10	172	64	83	249	370
31	Furniture	22,672	39	27	44	5	68	38	45	130	141
32	Other manufacturing	11,356	56	35	77	12	395	65	61	158	173
33	Repair of machinery	6,275	69	49	89	12	306	96	68	258	143
	Total Manufacturing	467,695	75	40	172	50	6,838	95	71	413	399

Source: Our elaboration. Productivity values are presented in BRL 1K. Cross-section values and firm averages are weighted by each firm observation. The sectoral average is calculated by the total value added divided by the sectoral number of employees.

by a decrease in market power and, thus, prices, or if it has negatively affected physical productivity as well. The second period represents the commodities boom, with metal ores and refined petroleum gaining a huge importance for the economy, and other sectors following the opportunities of the emergence of a new middle class, mainly in the northeast region of the country.

The data shows a meaningful intersectoral heterogeneity, with some sectors having great productivity but most being much less prolific, which makes Total Manufacturing to have a poor overall result, and an almost extreme tail. Our results are in concordance with other recent studies regarding Structural Heterogeneity for Brazilian Manufacturing (CATELA *et al.*, 2015). These facts provide sound evidence for the ECLAC Tradition of Centre-Periphery (PREBISCH, 1981; CIMOLI; PORCILE, 2013). The Structural Heterogeneity Approach advocated by ECLAC assumes that underdeveloped countries, particularly those in Latin America, have a hard-cut division between sectors that are well-integrated in International Trade[19] and those that are only competitive in the National Market, against a soft decay found for most developed countries. Now, we will improve this view to show that this intersectoral heterogeneity at the same time creates fairly similar productivity distributions among sectors.

Following Dosi *et al.* (2012), Yu *et al.* (2015b), Mathew (2017), we proceed to test the parametric fit of the productivity distributions using the Asymmetric Exponential Power distribution (AEP). The AEP distribution, introduced by Bottazzi and Secchi (2011), belongs to a family of distributions started by Subbotin (1923), which assumes a normal or Laplacian shape accordingly with the values of the b coefficients used, with values $b = 1$ generating a Laplacian, and values $b = 2$ generating a normal distribution. This distribution estimates the values of b for each tail independently, so b_l represents the coefficient for the left tail, while b_r represents the right one.

We estimate the fit of these parameters for the natural logarithm of productivity for each sector, which in turn will produce lognormal and log-Laplacian fits. We used a maximum likelihood method, but we were unable to achieve convergence for all sectors. The results are detailed in Table 6.

Somewhat more intensevely than expected, the AEP estimation reveals tails significantly fatter on the left side (particularly Food and Wearing). In fact, they are even fatter than what a log-Laplacian distribution would produce, and the estimates are smaller than the ones found for China and Italy (YU *et al.*, 2015b; DOSI *et al.*, 2012). More in accordance with the international results, the right side presents a steeper decline, very close to a lognormal distribution, with few exceptions.

Figure 12 shows the distributions of (log) productivity with the parametric fits of (log) normal and (log) AEP fits. AEP fits seems rather good and superior than the one produced by a (log) normal. It also seems very robust to different time periods and sectors.

The overall picture provides supporting evidence from what Dosi *et al.* (2012) called an “efficiency frontier”. Firms that are at the top of productivity in their sectors face constraints that are technological in their nature, which in turn create barriers for increases in productivity that are similar for all leaders, with far fewer outliers. Firms are more widely dispersed at the “bottom” of productivity, since their survival may be more attached to spatial or contextual advantages. Alternatively, their low productivity may reflect not a low physical productivity *per se*, but a low capacity to capture market earnings, and their

[19] Unfortunately, there is no microdata regarding exports by firms publicly available that would allow us a more profound exploration on the topic. Of course, the data is collected, but only available in aggregated representations.

Table 6 – Subbotin (AEP) Coefficients for Productivity in Brazilian Manufacturing - Cross-Sectional Data

ISIC	Industry	Labor Productivity			
		b_l	$\sigma(b_l)$	b_r	$\sigma(b_r)$
7	Metal ores	NA	NA	NA	NA
8	Other mining	0.87	(0.02)	2.57	(0.10)
9	Mining support	NA	NA	NA	NA
10	Food	0.55	(0.01)	3.35	(0.04)
11	Beverages	0.69	(0.02)	2.06	(0.07)
12	Tobacco	NA	NA	NA	NA
13	Textiles	0.62	(0.01)	3.58	(0.08)
14	Wearing	0.56	(0.00)	2.14	(0.02)
15	Leather	0.62	(0.01)	2.38	(0.04)
16	Wood Manufacturing	0.63	(0.01)	2.56	(0.05)
17	Paper	0.71	(0.01)	2.11	(0.05)
18	Printing	0.76	(0.03)	2.78	(0.11)
19	Refined petroleum	0.83	(0.03)	1.82	(0.10)
20	Chemicals	1.06	(0.02)	2.55	(0.07)
21	Pharmaceutical	NA	NA	NA	NA
22	Rubber and plastic	0.80	(0.01)	2.18	(0.04)
23	Other non-metallic	0.58	(0.01)	2.35	(0.03)
24	Basic metals	0.79	(0.02)	2.65	(0.08)
25	Fabricated metal	0.75	(0.01)	2.49	(0.04)
26	Computer and electronic	0.79	(0.02)	2.23	(0.07)
27	Electrical equipment	0.84	(0.02)	2.42	(0.07)
28	Machinery	0.99	(0.02)	1.92	(0.04)
29	Motor vehicles	0.87	(0.02)	2.29	(0.06)
30	Other transport	0.72	(0.03)	3.11	(0.17)
31	Furniture	0.57	(0.01)	3.17	(0.06)
32	Other manufacturing	0.61	(0.01)	3.25	(0.09)
33	Repair of machinery	0.85	(0.03)	1.79	(0.07)
	Total Manufacturing	0.63	(0.00)	3.05	(0.01)

Source: Our elaboration. b_l and b_r represents the left and right tail, respectively, while $\sigma(b)$ represents the standard deviation of the estimated parameters.

adverse positioning in their production network (STURGEON, 2002; GEREFFI *et al.*, 2005), especially if they are producing for intermediate consumption, which may make them captives of the monopsony power from the leading firms.

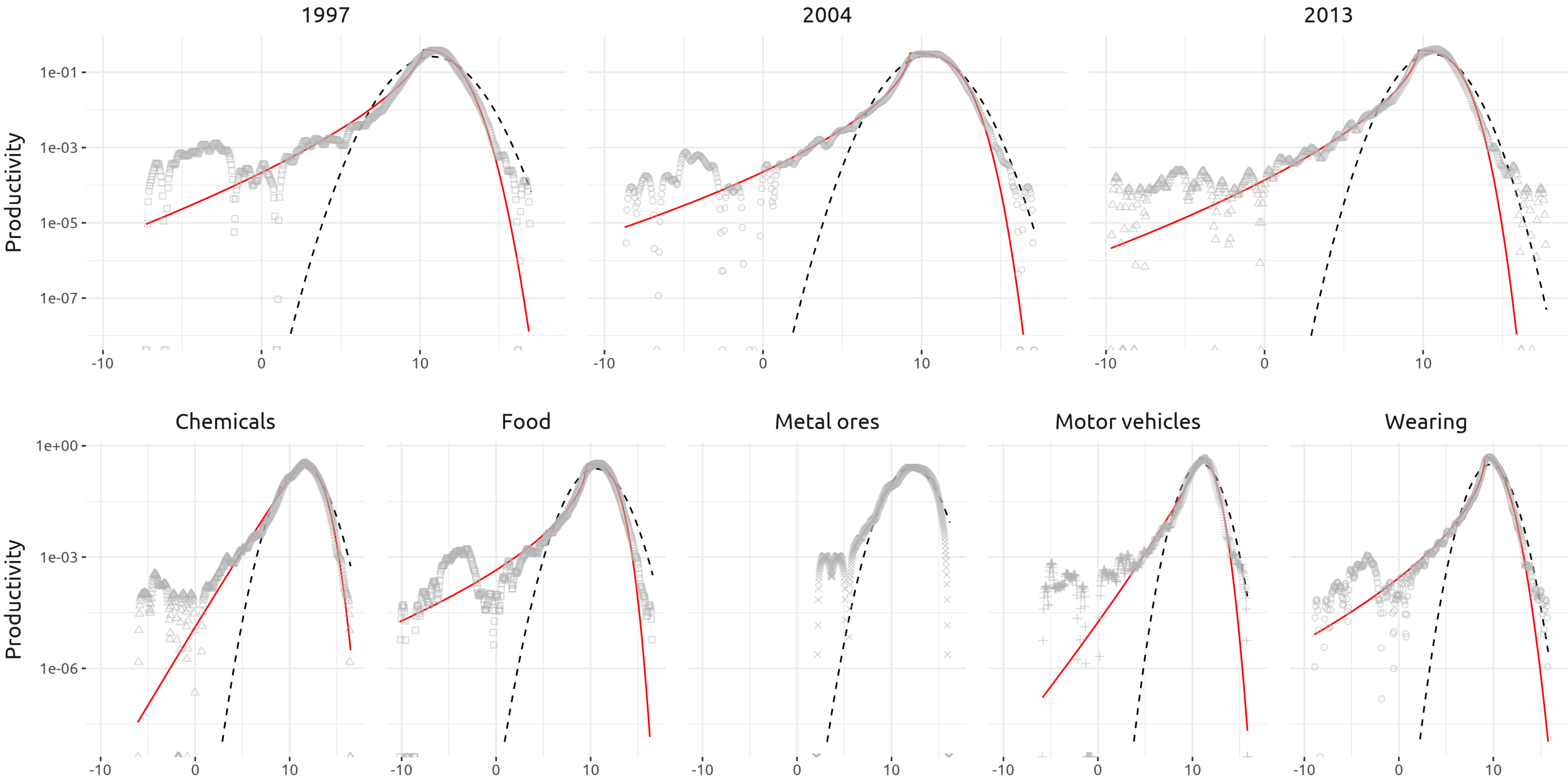


Figure 12 – Log Labor Productivity - Probability Density Plots. Dashed lines represent a normal fit for each distribution, while the red lines represent the AEP fit.

Rates Distributions

In this section, we analyze the nature of the distributions of firm growth rates and productivity change. These variables are fundamental to understand the economic process as they are the power that shape the markets. In fact, there is no capitalism without dynamics. To understand them, thus, is to understand how market and customers interact to decide whom are the ones that will be chosen to produce and what will keep being produced.

It is remarkably intriguing that this process would ever assume any particular shape. There is no obvious reason why dynamics should have to follow a particular mechanism or to be able to be modeled by simple stochastic principles. However, as imaginative as nature is, such patterns do exist and have now been found for several countries.

We begin our investigation by looking at the basic statistics of each variable. Table 7 presents cross-sectional values for the moments of each proxy. Notice that we lost close to a hundred thousand observations due to firms that were not present in any of the years of the survey. Growth has a positive average rate for all but a few sectors, and productivity change shows mostly a insignificant or negative result. Only one sector shows a negative median growth, while productivity change presents 14 sectors with a weak or negative result. Both metrics present very fat tails for the majority of sectors.

Growth is quite symmetrical for most sectors, with few exceptions. Productivity change is more asymmetrical, with sectors presenting negative skewness, an unexpected result. That means that for these sectors, most firms kept their productivity, with a few outliers pushing the distribution downwards, what is also elucidated by the median greater than the average.

Sectoral values present the accumulated variation in the period for total revenue and productivity, calculated as the difference of the simple size and productivity averages of 1996 and 2013[20]. Mining support (ISIC 9) and other transport (ISIC 30), as seen previously, have a remarkably growth, followed by repair of machinery (ISIC 33), and in a wide distance, metal ores (ISIC 7) and refined petroleum (ISIC 19). This result does not necessarily translate itself in higher sectoral productivities. From those sectors, only other transport (ISIC 30) has an increase in sectoral productivity.

[20] In other words, we calculated the difference of the sectoral total revenue of the two periods for size growth, and the difference of sectoral value added over total employees for productivity.

Table 7 – Firm Growth and Productivity Change in Brazilian Manufacturing - 1996-2013

ISIC	Industry	Total Obs.	Δ% Tot. Rev.						Δ% Prod.					
			Avg.	Median	Sd.	Skew.	Kurt.	Sectoral	Avg.	Median	Sd.	Skew.	Kurt.	Sectoral
5	Coal and lignite	206	−1	−2	26	0	8	37	−5	−4	60	−2	20	−7
6	Crude petroleum	39	10	11	50	1	6	NA	23	6	75	1	4	NA
7	Metal ores	744	7	4	51	1	8	155	2	0	87	0	16	−11
8	Other mining	6,734	3	3	40	0	9	84	1	2	118	−2	59	3
9	Mining support	492	7	2	45	0	5	1,685	−1	−4	72	0	11	−16
10	Food	43,208	4	4	42	0	21	54	0	1	155	0	58	−23
11	Beverages	5,208	4	3	42	0	20	122	0	1	166	−1	42	32
12	Tobacco	485	−4	0	60	0	8	−68	−3	−3	89	0	7	−59
13	Textiles	16,792	2	2	37	−1	47	6	0	1	121	−1	82	4
14	Wearing	41,119	3	1	44	0	20	46	2	1	127	0	63	−24
15	Leather	20,874	1	0	43	0	12	−15	1	0	98	0	88	−41
16	Wood Manufacturing	15,599	0	0	45	0	10	51	−1	0	114	−1	54	41
17	Paper	10,490	3	3	34	0	16	61	1	1	137	1	82	21
18	Printing	4,188	3	2	40	0	31	22	2	1	95	3	144	−45
19	Refined petroleum	2,777	6	5	45	2	29	152	0	0	128	0	68	38
20	Chemicals	17,234	3	3	35	0	19	37	−1	−1	93	0	54	−6
21	Pharmaceutical	3,991	5	4	31	0	13	62	0	1	71	−1	62	−5
22	Rubber and plastic	27,502	2	2	36	0	20	54	−1	−1	119	0	67	−16
23	Other non-metallic	28,330	2	2	37	0	17	31	0	0	86	0	37	−25
24	Basic metals	8,883	3	3	38	1	15	41	0	0	101	−1	77	−1
25	Fabricated metal	30,410	4	3	43	0	22	104	1	1	103	−1	53	0
26	Computer and electronic	8,372	4	4	44	3	84	66	−1	1	119	0	81	1
27	Electrical equipment	10,643	3	3	39	0	15	51	−1	0	93	−2	51	−12
28	Machinery	25,582	2	2	39	0	14	69	−1	−1	81	−1	70	2
29	Motor vehicles	14,025	3	2	36	0	16	101	0	−1	98	0	81	17
30	Other transport	2,769	7	7	58	0	26	507	1	3	114	−2	61	48
31	Furniture	18,047	3	3	39	−1	49	86	1	2	128	0	41	8
32	Other manufacturing	9,117	3	3	39	2	151	82	2	1	98	0	60	10
33	Repair of machinery	4,358	5	4	50	0	12	468	4	2	103	1	70	−45
	Total Manufacturing	378,218	3	2	40	0	26	67	0	0	116	0	71	−3

Source: Our elaboration.

The results for the AEP estimates are presented in Table 8. They show that distributions on growth rates and productivity change are fairly symmetrical for most sectors, with all the b estimates close to one or below, indicating tails that are at least Laplacian. Growth rates present values that are comparable with the ones from India (MATHEW, 2017) and lower than the ones found for Italy and US (BOTTAZZI; SECCHI, 2003; BOTTAZZI *et al.*, 2007), while productivity change presents values lower than those found for China (YU *et al.*, 2015b).

Table 8 – Subbotin (AEP) Coefficients for Firm Growth and Productivity Change in Brazilian Manufacturing - Cross-Sectional Data

ISIC	Industry	Δ% Tot. Rev.				Δ% Prod.			
		b_l	$\sigma(b_l)$	b_r	$\sigma(b_r)$	b_l	$\sigma(b_l)$	b_r	$\sigma(b_r)$
7	Metal ores	0.77	(0.06)	0.80	(0.06)	0.65	(0.05)	0.82	(0.06)
8	Other mining	0.97	(0.03)	0.99	(0.03)	0.62	(0.01)	0.63	(0.02)
9	Mining support	0.92	(0.12)	1.21	(0.14)	0.92	(0.10)	0.84	(0.09)
10	Food	0.72	(0.01)	0.78	(0.01)	0.51	(0.00)	0.57	(0.01)
11	Beverages	0.86	(0.03)	0.85	(0.03)	0.50	(0.01)	0.51	(0.01)
12	Tobacco	0.67	(0.06)	0.80	(0.08)	0.92	(0.10)	0.80	(0.08)
13	Textiles	0.88	(0.02)	0.88	(0.02)	0.54	(0.01)	0.59	(0.01)
14	Wearing	0.78	(0.01)	0.91	(0.01)	0.54	(0.00)	0.61	(0.01)
15	Leather	0.87	(0.01)	0.90	(0.01)	0.63	(0.01)	0.68	(0.01)
16	Wood Manufacturing	0.86	(0.02)	0.92	(0.02)	0.66	(0.01)	0.69	(0.01)
17	Paper	0.81	(0.02)	0.82	(0.02)	0.56	(0.01)	0.52	(0.01)
18	Printing	0.85	(0.03)	0.79	(0.03)	0.72	(0.02)	0.62	(0.02)
19	Refined petroleum	0.95	(0.04)	0.77	(0.03)	0.64	(0.02)	0.66	(0.03)
20	Chemicals	0.82	(0.01)	0.85	(0.01)	0.60	(0.01)	0.66	(0.01)
21	Pharmaceutical	0.84	(0.03)	0.83	(0.03)	0.58	(0.02)	0.70	(0.02)
22	Rubber and plastic	0.81	(0.01)	0.85	(0.01)	0.55	(0.01)	0.55	(0.01)
23	Other non-metallic	0.90	(0.01)	0.91	(0.01)	0.65	(0.01)	0.69	(0.01)
24	Basic metals	0.96	(0.03)	0.83	(0.02)	0.62	(0.01)	0.67	(0.01)
25	Fabricated metal	0.81	(0.01)	0.92	(0.01)	0.57	(0.01)	0.65	(0.01)
26	Computer and electronic	0.89	(0.02)	0.82	(0.02)	0.61	(0.01)	0.60	(0.01)
27	Electrical equipment	0.81	(0.02)	0.99	(0.02)	0.59	(0.01)	0.65	(0.01)
28	Machinery	0.90	(0.01)	0.97	(0.02)	0.67	(0.01)	0.69	(0.01)
29	Motor vehicles	0.85	(0.02)	0.97	(0.02)	0.59	(0.01)	0.58	(0.01)
30	Other transport	0.72	(0.03)	0.79	(0.03)	0.57	(0.02)	0.71	(0.03)
31	Furniture	0.82	(0.01)	1.01	(0.02)	0.51	(0.01)	0.59	(0.01)
32	Other manufacturing	0.86	(0.02)	0.88	(0.02)	0.58	(0.01)	0.66	(0.01)
33	Repair of machinery	0.84	(0.03)	0.90	(0.03)	0.66	(0.02)	0.64	(0.02)
	Total Manufacturing	0.82	(0.00)	0.88	(0.00)	0.57	(0.00)	0.61	(0.00)

Source: Our elaboration. b_l and b_r represents the left and right tail, respectively, while $\sigma(b)$ represents the standard deviation of the estimated parameters.

Figure 13 presents the distribution of growth rates and productivity change for three years (notice the log-transformation in the vertical axis), together with both AEP and normal fits. The graph for each period and proxy shows a very clear, ‘tent like” shape. Also, notice the poor fit of the Normal distribution to describe the tails. The normal fit falls much faster than the empirical rates, which demonstrates, as the kurtosis already signaled, that infrequent events of extreme impact are much more “common” than it would be expected under normality.

Figure 14 shows the same plot for five different sectors. A similar “tent-like” shape as before is found, proving this pattern to be robust under disaggregation. Specifically, productivity change for sectors Food (ISIC 10) and Wearing (ISIC 14) presents some symmetrical inflections at both ends of the distribution that deserve more investigation. Overall, the “tent” shape is very solid, and characterizes a Laplacian curve.

This shape demonstrates the presence of some kind of short-run correlation between the events that produced growth. The fact that the empirical long-run growth distributions converge very slowly to a normal shape attests that these correlations survive to time frames longer than a year. Bottazzi and Secchi (2006a) explored in detail this phenomenon in a model following the Simon’s tradition of “islands of opportunity”(IJIRI; SIMON, 1977). If we suppose that there is a limited availability of growth episodes available for firms, and that the ones that took these opportunities in the past have more chance of winning them in the future, thus generating a path-dependent mechanism of competition, then the model is able to reproduce asymptotically this Laplacian shape.

At the same time, these distributions give an interesting contrast with some notions from innovation theory. First the notion of capabilities, which are incremental in the sense that they are hard to obtain, and must be accumulated and built upon, thus constituting the core of value generated by the firms (PENROSE, 1959; MALERBA, 1992; TEECE *et al.*, 1997; GEREFFI *et al.*, 2005) , with learning by doing being an important factor (ARROW, 1962). Second, the idea of technological trajectories, which are mostly subject to periods of incremental improvement with discontinuities following structural breaks due to radical or disruptive innovation (DOSI, 1982; DOSI; NELSON, 2010). These two concepts, together, would make one expect for fairly smooth periods of incremental perfecting followed by large jumps of rapid growth due to change of paradigms.

Instead, the shape of growth or productivity change rates distributions is constantly bombarded by a process that generates extreme, symmetrical events. It sounds implausible that in all these cases some disruptive innovation is happening for a few, and not necessarily the same, enterprises all the time, notably for sectors that are already mature or stagnate. So, stochastic and simpler models as the ones proposed by Bottazzi and Secchi (2006a) seem closer to the empirical data.

This, of course, does not disavows any theory of incremental innovation or continuous improvement, but suggests that there are important middle steps between what configures learning in the sense of technological advancement and organizational management and what in fact generates financial returns, the latter being somewhat more extreme in its deviation, and at the same time, relatively constant in its nature. Increases in physical productivity doesn’t necessarily translate themselves in increased monetary productivity, and quality change doesn’t imply sale growth. Especially with products with high standardization, a lot of these gains became customer surplus, e.g., transistors and steel

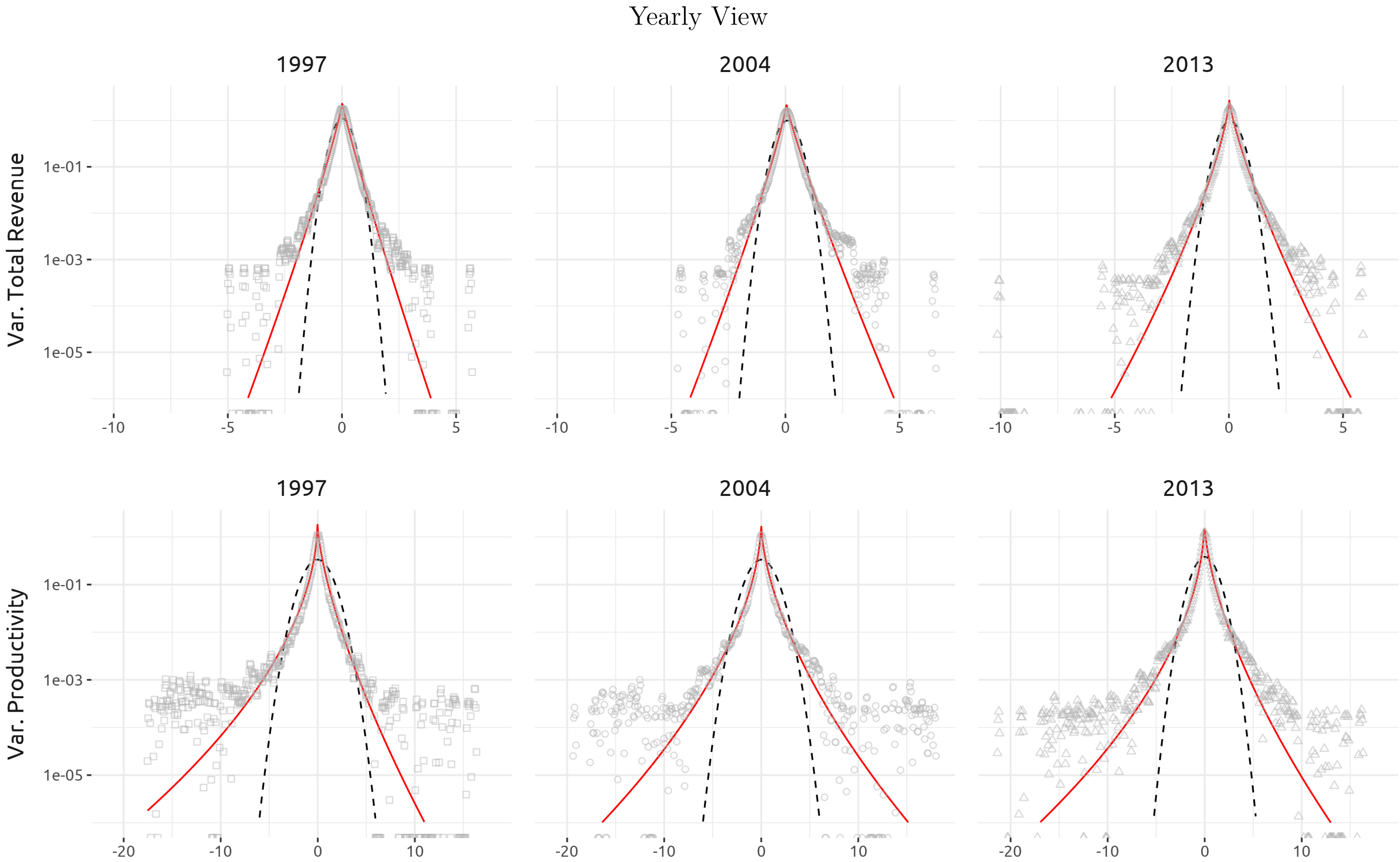


Figure 13 – Growth and Productivity Change - Annual Probability Density Plots. Note the vertical axis in natural logarithm. Dashed lines represent a normal fit for each distribution, while the red lines represent the AEP fit.

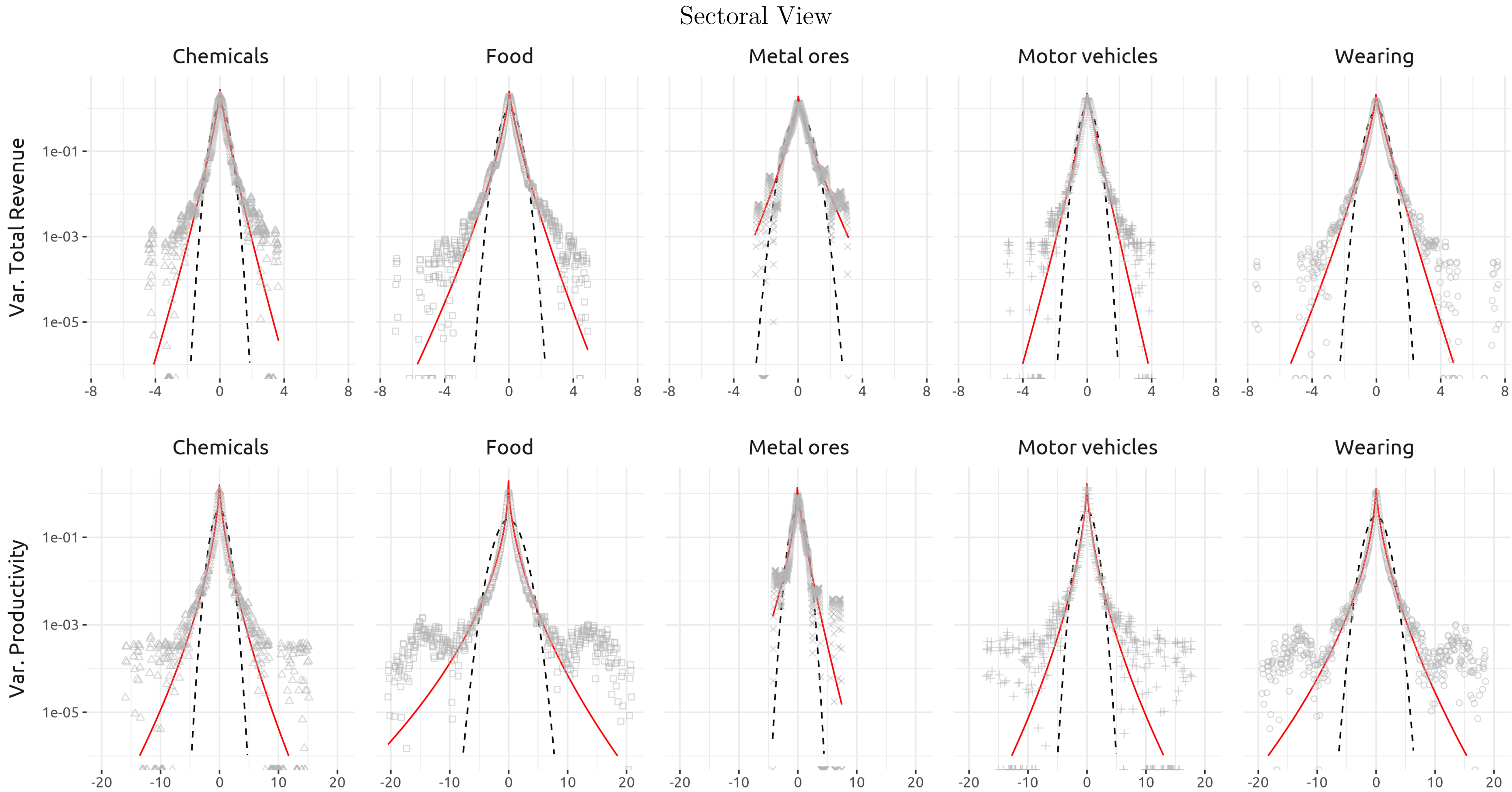


Figure 14 – Growth and Productivity Change - Sectoral Probability Density Plots. Note the vertical axis in natural logarithm. Dashed lines represent a normal fit for each distribution, while the red lines represent the AEP fit.

production (DOSI; NELSON, 2010).

So, while we are obviously not disagreeing with the idea that physical productivity and technology change play an important role in monetary growth and monetary productivity change, these theories must be adapted to faithfully incorporate the kind of short-term competition and the network nature of markets, such as to define who is more probably to take the gains of innovation: costumers, leaders or innovators. (GEREFFI *et al.*, 2005).

2.4 Conclusion

This chapter presented a list of statistical measures regarding Brazilian Manufacturing. Our results corroborate the international literature and weights favorably to the hypothesis that the stylized facts on growth, productivity and size may describe timeless economic phenomena.

Among them, our highlights are the 1) ubiquitous heterogeneity found in the most important economic proxies for size, performance and growth; 2) the skewness of firm size distributions, well described by lognormal and Pareto distributions; 3) the efficiency frontier and the roles that hierarchies may play in the productivity distributions, and finally 4) the Laplacian shape of firm growth rates and productivity change, implying some type of short-term correlation and competition across business opportunities.

Our interpretation of these results is that they move us to a more complex representation of the markets than what is usually thought. At the same time, the periodicity and robustness of these stylized facts put the theorist in a much firmer ground. In fact, we feel that this kind of characterization of empirical results in stylized facts helps more to advance the field than oblivious testing of *a priori* hypotheses. In this sense, we follow the spirit of the words of Gabaix (2009, p. 285): “Estimate, don’t test!” and Tukey (1962, p. 13), where “it is better to have an approximate answer to the right question, which is often vague, than an exact answer to the wrong question, which can always be made precise”. So, what are the consequences of these results for the economic profession?

We think that the current stream of empirical evidence regarding both industrial organization, behavioral economics, labor markets and the most useful tools developed by the great empiricists of the XXth century, a.k.a, Wassily Leontieff, Colin Clark and Simon Kuznets, formally as the National Accounts, needs a deep integration with models that can adequately reproduce what is empirically found while having dept in economic thought. The class of models broadly named as “Schumpeter meets Keynes” (DOSI *et al.*, 2010) is a valid effort in this direction, but one that is only at the beginning. The network nature of economics must be recognized, and we need to develop a more realistic representation of the intermediate expenditure, e.g. drawing in the literature of complexity (HAUSMANN; HIDALGO, 2014; HARTMANN *et al.*, 2017), such as input-output tables at the firm level, which will enable to enrich representations as the ones developed by

Gereffi *et al.* (2005)..

So, the overall prognostic is optimistic, and while our knowledge of economics will always probably be only of a statistical nature, the lack of data and computational power that affected the previous generations are not a problem anymore, and nowadays they give us the opportunity to bring the economic field to more firm grounds.

3 Survival of the fittest or does size matter? Empirical evidence on market selection and size for Brazilian Manufacturing and Services

This article aims to explore learning and selection effects of productivity change for three classes of firm sizes in Brazilian manufacturing and service sectors from 1996 to 2011. The methodology is based on the Price Equation, a variance decomposition method. Our results support the international evidence about the weakness of the selection effect to explain aggregate productivity change for medium and large size firms. Small firms, however, are much more affected. Size, measured by number of employees, appears to be a good proxy for capital intensity. There are as well signs that the learning effect is highly correlated with the economic cycle.

JEL: L11, D22, L60, L80.

3.1 Introduction

The importance of productivity in economic models is unquestioned. From deriving a market equilibrium and determining international comparative advantages to describing market evolution in neo-Schumpeterian models, it plays a central role governing the market dynamics. But so little yet is known about the mechanism that promotes aggregate productivity change.

An important turning point on the discipline was the growing availability of micro-level data with a systematic representation of industry at the firm level. By having the appropriate information of profitability, productivity and corporate growth, the data allowed new insights on the understanding of market functioning.

This led to numerous studies evaluating the transformation of productivity using decomposition methodologies and parametric estimations. The great heterogeneity found regardless of the level of disaggregation, and especially, its persistence through time, created unpleasantness with the concept of aggregate production functions. It also exposed the weakness of averages taken from sectoral level analysis as they largely simplify the underlying phenomena.

Further, the great turmoil of entry and exit of firms seemed to fit well with a Schumpeterian view of *creative destruction*. Differently from the neo-classical perspective, where firms enter and exit the market only to reestablish the equilibrium of market's price, the idea of creative destruction assumes a constant process of renovation, with lots of churning and where new firms consistently replace the fallen ones.

In this sense, the relationship between growth and productivity, given by different families of theoretical models, usually involves more productive firms gaining market-share either by lowing mark-up or through larger investments driving more innovation, better products and processes. A first approach is given by what was called an "evolving equilibrium" or "dynamic equilibrium", and it is exposed in works that embed heterogeneity as a fundamental force, like Jovanovic (1982), Hopenhayn (1992), Ericson and Pakes (1995), Olley and Pakes (1996), Luttmer (2007) and Acemoglu *et al.* (2013). Another approach is given by the neo-schumpeterian literature, with the classic from Nelson and Winter (1982) and others like Winter (1984), Silverberg *et al.* (1988), Dosi *et al.* (1995), Silverberg and Verspagen (1995), Metcalfe (1998), Winter *et al.* (2000), Winter *et al.* (2003), and the most recent family of agent-based models called "Schumpeter meeting Keynes", which provide macro models with empirical microfoundations (DOSI *et al.*, 2010; DOSI *et al.*, 2017).

The necessity of measuring this dynamics led to a rich route of decomposition methodologies. This article contributes to this literature in three important ways: 1) it covers both industry and services, giving a broader context of Brazilian economy; 2) it uses the Price Equation as the decomposition method, a still underexplored tool to describe evolutionary change of any type; 3) it performs a decomposition analysis considering three categories of firm's size, allowing better clarity in the characterization of these results for both segments.

Our main outcomes suggest that, confirming what was found in the international literature, the selection forces acting upon market are not as strong as what was initially thought. The idiosyncratic learning process inside the firms seems to play a larger role in aggregate productivity change. But this doesn't tell the whole story. There are significant changes accordingly with firm size, as measured by number of employees. Smaller firms productivity appears to be much more affected through selection than the larger ones. Also, although the absolute values are greater for the learning effect, it is hard to point a defined trend, and their signals seem highly correlated to the economic cycle.

3.2 Background Literature

Besides the topic of productivity being extensively explored throughout the twentieth century[1], the first studies using modern micro-level data appeared only in the early

[1] Salter (1966) is an earlier example of the kind of analysis conducted here.

nineties. Baily *et al.* (1992) was one of the pioneers to describe the relationship between productivity and market composition for the US Manufacturing. Other studies for US were conducted by Baily *et al.* (1996), which find great heterogeneity among firms regardless of the disaggregation level, and Bartelsman and Dhrymes (1998), which demonstrates its high persistence through time. Similar studies were also conducted for other countries, like Israel (GRILICHES; REGEV, 1995), United Kingdom (DISNEY *et al.*, 2003a; DISNEY *et al.*, 2003b), Germany (CANTNER; KRUGER, 2008), Chile (PETRIN; LEVINSOHN, 2012) and Canada (BALDWIN; GU, 2006).[2]

Among several stylized facts that these studies analyze, we find a minor role for the selection effect - the reallocation of shares among continuing firms[3] - with most of productivity change being caused by the movement of entry and exit of firms and due to internal variation. Parametric estimations of the relationship betweeng growth and productivity also corroborated these results. Dosi *et al.* (2015), improving on Bottazzi *et al.* (2010), found a small contribution of selection for France, Germany, UK and US, with most of the impact of productivity on growth coming from the first difference of relative productivity - that is, the variation of the distance of each firm's productivity from the average productivity - rather than in relative productivity by itself, or the distance of each firm's productivity from the average. Analogous results are found in Chinese Manufacturing by Yu *et al.* (2015a).

Another fact that usually appears in the empirical studies is the heterogeneity among firms and the most diverse variables analyzed. Apart from the previous literature, heterogeneity was extensively investigated. Such analysis produced as a stylized fact a fatter left tail for productivity distributions, with smaller firms more dispersed than the bigger ones, which indicates some kind of "efficiency frontier", and a characteristic Laplacian format for productivity change, which resembles a "tent shape", robust to all degrees of disaggregation available in different countries (BOTTAZZI *et al.*, 2007; BOTTAZZI; SECCHI, 2003; BOTTAZZI; SECCHI, 2005; YU *et al.*, 2015b).

Finally, other research is related to the relationship of productivity with size. In general, even if the evidence is more dubious for smaller firms (LOTTI *et al.*, 2001), growth does not seem to be correlated either with productivity or profitability (BOTTAZZI *et al.*, 2010; YU *et al.*, 2015a). On the other hand, size and productivity are important metrics for survival, where smaller and less productive firms die faster (BAILY *et al.*, 1992; GRILICHES; REGEV, 1995) and entry and exit are highly correlated, with sectors with a high number of entrants usually having a high number of exiters. That is, markets are relatively stable in size (DISNEY *et al.*, 2003a).

One main issue that all these studies consider is the methodological one, on how to measure those variables as well as the choice of the proxies to use for that purpose. The

[2] For two reviews of the literature see Bartelsman and Doms (2000) and Foster *et al.* (2001).

[3] Some studies, like Disney *et al.* (2003a), even find a negative value for this component, suggesting a reallocation to less productive firms.

diversity of methodological possibilities in this venue is not trivial. For example, if we consider productivity as an efficiency index, in its basic conception it is given by the input-output relationship, which bring questions on how to measure inputs (e.g., number of workers, number of hours) and outputs (e.g. gross revenue, value added).

In general, it is possible to measure productivity by Total Factor Productivity (TFP) or labor productivity. TFP summarizes the complex network of tasks used to create value in two specific inputs, capital and labor, thus controlling for changes in productivity related to the quantity of each factor as much as technological change.

Unfortunately, this forces the adoption of very restrictive hypothesis about the relationship between labor and capital, and the conditions that allow the use of aggregate production functions for macro analysis of sectoral or global productivity are so stringent (FISHER, 2005), that their use should be carefully considered. Additionally, they are inconsistent with the evolutionary theory used as theoretical reference in this paper.

In the authors opinion, the idea of substitution between capital and labor in any modern industry seems implausible, with the exception of the most basic tasks. Leontief-style production functions, pragmatically, describe most of the relationship between labor and capital. On the empirical side, there are problems regarding the estimation of these metrics since they represent a mathematical transformation of accounting identities (FELIPE; MCCOMBIE, 2013). Finally, our data on firm capital is also not reliable, being unrepresentative of the whole sample, with missing values in about half of the total observations for manufacturing.

Nevertheless, since other studies find a high level of correlation between both labor productivity and TFP (FOSTER *et al.*, 2001), we don't think that the use of the former, which we choose, should constitute a bias in our results in the view of other schools of economic thought.

Labor productivity is usually measured as a relation between value added or gross revenue per employees or hours worked. Gross revenue suffers from impacts of price changes in intermediary inputs, so we opted to use value added per employee as a more robust proxy of the internal factors that affect productivity. We also consider that, four our study, number of employees is a better proxy than number of hours worked, since the last one is probably more efficiently tracked for bigger than smaller firms, which would constitute a size bias in our reports.

Other issue is related on how to correctly address the importance of each firm in the aggregate index. This weight is usually measured by proxies of firm size, with total revenue or employment share being the most common ones. For this work, we chose the employment share, since again, it represents a factor that is internal to the firm, and is not affected by intermediary consumption.

In this work, the method adopted for the productivity decomposition is the Price Equation. It was developed by George R. Price to study inheritance of genetic traits in

Biology, but as he posed, it is easily generalized to deal with any characteristic that evolves with time, in any field (FRANK, 1995; PRICE, 1970; PRICE, 1972; PRICE, 1995). The Equation, very cleverly, clarified the relationship expressed in the Fisher's Fundamental Theorem of Natural Selection (FISHER, 1930), and enabled a merge between Darwin's evolution and John Nash's work in Game Theory. It is important to notice that the structure and formulation of the Price equation is not attached to any theoretical - and particularly biological - content. Its structure, when compared to other decomposition approaches, has as the main advantage the possibility of performing a multilevel analysis, which has already been used in Holm (2010) and lately in Luna *et al.* (2015), for the analysis of the Danish and Brazilian industry, respectively. This multilevel analysis allows the characterization of the selection effect in all the current and subordinate structures, as, for example, selection occurring among different sectors and selection occurring among firms in the same sector.

Furthermore, as Holm (2010) describes, there are other theoretical works in which it plays a central role: in determining evolution of routines' frequencies, such as in a Generalized Darwinism perspective (ANDERSEN, 2004; HODGSON; KNUDSEN, 2004); in neo-schumpeterian models, where it appears as a mathematical expression for the construction of evolutionary explanations in line with the replicator's dynamics (METCALFE, 1994; METCALFE, 1998; METCALFE; RAMLOGAN, 2006), and in the general principle of selection of evolving systems (KNUDSEN, 2004).

Other decomposition methods frequently used in the literature are the modified version of Baily *et al.* (1992), proposed by Foster *et al.* (2001), and the method of Griliches and Regev (1995). The Price Equation resembles the first, with the difference being that it doesn't separate the within effect into a cross-variance effect and a constant-share learning effect. The method of Griliches and Regev is similar in this respect, but uses an average of the shares between periods to prevent against measurement error. The entry and exit terms are related in all, with minor differences in the variables regarding the use of initial or final period values.

Notwithstanding this, it is difficult to compare results amidst them or to use different methodologies to test for robustness, as it may be the case that all are valid *per se* and ultimately are measuring distinct things (HOLM, 2010).

It is important to note that this work is far in analytical and methodological terms of other similar studies for Brazil, especially those departing from traditional aggregate production functions, such as the ones presented in Ferreira *et al.* (2008) and in Bonelli and Bacha (2013), among others. Despite the existence of several works on productivity of the Brazilian Manufacturing, the service sector is largely neglected. Also, as in the previous papers cited, the explanations behind the industrial dynamic are not under an evolutionary framework. Recently, and in an evolutionary context, Catela *et al.* (2015) presents a non-parametric approach for the analysis of the evolution of sectoral labor productivity and its determinants, for the period of 2000-2008 for the Brazilian Manufacturing. The results put in evidence the existence of market asymmetries, showing that few sectors have a high labor productivity and that less productive sectors are more heterogeneous, which is also verified through quantile regressions. Hence, the comprehension of the Brazilian industry dynamic under an evolutionary point of view shows a necessary and promising venue of research.

This work aims to complement this literature with a descriptive analysis of the evolution of labor productivity change, considering the impact of size on the performance of firms and with the evolutionary theory to lighten and explain our findings.

3.3 Data

This study is based on two databases from IBGE, the Brazilian Institute of Geography and Statistics, responsible for collecting and publishing most of statistical data of the country: PIA[4],which is the annual survey of the manufacturing sector (ISIC Codes 10-36) and PAS[5],which is its mirror for the service sector (ISIC Codes 55-93 plus services related with agriculture and livestock).

Both databases have census information for firms over 20 (PAS) and 30 (PIA) employees from 1996 to 2011 in the case of the manufacturing sector and from 1998 to 2011 for the service sector. The access to the data is restricted and due to privacy reasons we are commited to exclude any sector with less than 03 firms. It is important to highlight that both databases only include information for the formal economy. This is more significant for services since Brazil, historically, has a great share of informal economic activity in this sector. Moreover, despite the relevance that small firms have on the Brazilian economy - especially in the service sector - it is important to remark that our sample is responsible for at least 65% and 80% of the whole added value of the service and manufacture sector, respectively (SEBRAE, 2014). For Manufacturing, value added was proxied by the value of industrial transformation[6] (VTI), while for the service segment the usual definition was employed.

The nominal values were deflated for the manufacturing sector with 2-digit sectoral prices indexes (IPA-OG). For the service sector, these indexes were not available, so we used a general aggregate index for all subsectors (IPCA-Geral).

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4 PIA - Annual Industrial Survey.

5 PAS - Annual Services Survey.

6 The differences of these two criteria are given by the Brazilian Statistical Office (IBGE) and occurs both in the revenues and costs considered. The value of industrial transformation (known as VTI) takes only the costs directly involved in production, such as raw materials, energy and maintenance, while the added value criterion also deducts rent, advertising, freight, among others. The same occurs in the revenues, where financial operations are removed and only income from products manufactured or inventory changes are included. Both are used indistinctly by IBGE.

For a better presentation of the economical and political context of the Brazilian economy from the late nineties to the early 2000's we opted to split the data in two periods. The first period, 1996-2003, represents the efforts of stabilization of the currency through a pegged system linked to the US dollar. It covers most of the period of the overvalued exchange rate and two international crisis, the first with the Balance of Payments Crisis of the Emergent Countries in 1997-1999, and the second with the burst of the Dot-com Bubble in 2001. In the whole period, these crises resulted in a low raise in value of transformation (11%) and employment (16%). Productivity in this context is mostly decreasing for manufacturing and, with the exception of a short recovery in 2001-2002, stagnant for services.

The second period, 2004-2011, represents the growth of the economy following the commodities boom. Several infra-structure projects were initiated in this time, and government investment was more fiercely achieved than in the previous period. Also, there was a strong growth in internal demand, based on consumption and on the increase of credit and wages. Manufacturing's value added and employment expanded 41% and 29%, respectively. Productivity for both manufacturing and services grow steadily.

Despite the fact that the Price equation can consider the effect of entry and exit, this

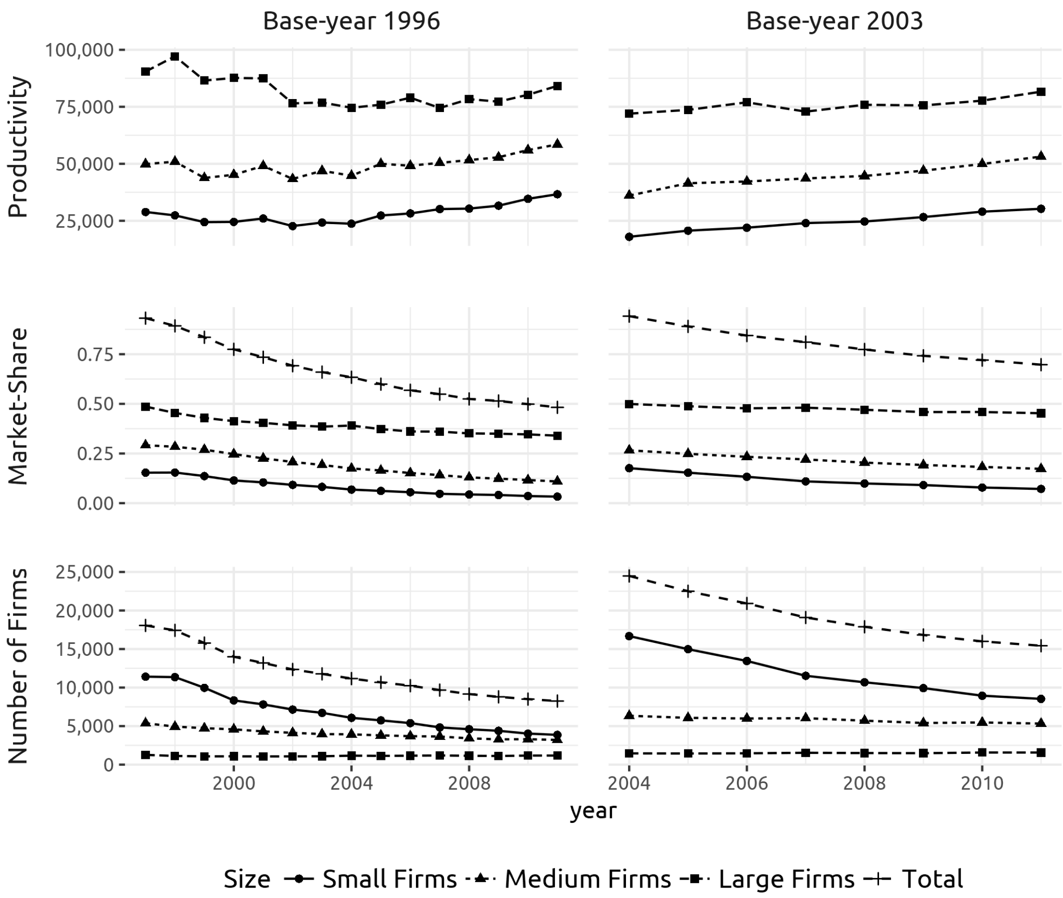

Figure 15 – Manufacturing - different metrics of incumbents by size.

work considers only the selection and learning effects. The focus on those two effects is due to the lack of access to another database[7] (RAIS), which would make possible the inclusion of age and other variables of interest. These variables would make viable a demographic study of the firms, which we plan to consider in future works.

Therefore, our investigation is based on the observation of incumbents, which are the firms that are present both in the base-year and any end-year of the panel. This means that our sample is not random, as numerous studies (DISNEY *et al.*, 2003a; BALDWIN; GU, 2006) highlight that size is negatively correlated with probability of exit. Nevertheless, the impact on medium and large size firms is very reduced, and the number of incumbents declines slowly.

Figures 15 and 16 show some selected variables related to incumbents according to their size, for both periods and sectors. As mentioned before, the size of a firm is defined by its number of employees and three categories are established. Small firms, with a number of employees between 30 (20 for services) and 99; medium firms, between 100 and 499 and large enterprises as the ones that have 500 or more employees. The panel for all sizes has around 20 thousand firms per year for each period. In addition to the number of firms by

Figure 16 – Services - different metrics of incumbents by size.

7 The RAIS database covers information about the formal employment of all firms in Brazil and is organized by the Brazilian Ministry of Labor.

size category, average productivity and market-shares for manufacturing and services are also depicted in both figures.

It is interesting to see the relevance of size in the market-share[8]. Large firms represents around half of the total employment for each sector among incumbents. Also, there is a huge discrepancy in productivity associated with size for manufacturing, and large firms present double and quadruple output per worker when compared to medium and small firms, respectively. This contrasts with services, where productivity is about the same for all sizes, thus indicating huge scale gains in the former. The magnitude of this productivity gap for the diverse size categories seems to be a particularity of Latin America, and our finding is strongly corroborated by other studies (CEPAL, 2010; SANTOLERI; STUMPO, 2016).

3.4 Methodology

Aggregate productivity growth is measured through a weighted average of the productivity of each firm. This growth is the result of the reallocation of shares of the market between incumbent, entering and exiting firms and changes in their productivity levels.

The productivity change of incumbent firms can be separated in two outcomes. The within effect corresponds to firm-specific variations in productivity levels and it is usually associated to the activity of idiosyncratic learning and innovation that occurs inside the boundaries of the firm. The between effect, on the other side, represents changes in the landscape of the market. It accounts for the gains and losses of market-share, weighted by the productivity of the firms, and represents a measurement of selection forces acting to promote the fitness of the environment. Both are also referred as a learning and selection effect, respectively, and this terminology will be used indistinctly along this article.

Let's start with an aggregate index of productivity, Z, the productivity of individual firms, z_i, measured as the logarithm of the value added per worker[9], and s_i the market share of the firm, measured as its participation in total employment:

$$Z = \sum_{i \in C} z_i s_i \tag{3.1}$$

[8] The gap between the total market share and 100% for each year is due to sectoral turbulence: entrants and exiters that are not considered in our sample, changes on the size of a firm below the census level (lower than 30 - or 20 for services), change on the firm activity or any other reason not specified in the database.

[9] The advantage to use a logarithmic expression in this case is that it makes sense that relative values of productivity would be more important than the absolute ones. On the other hand, this forces us to exclude all firms with a negative value added. These firms mostly represent fill-in errors, or firms that are in process of bankruptcy and were not yet excluded from the database. Their number represents a very small fraction of the total observations.

where C, represent firms that are incumbents. This aggregate index can then be decomposed using the Price Equation. The index variation is expressed as[10]:

$$\Delta Z = \sum_{i \in C} \Delta s_i (z_i - Z) + \sum_{i \in C} s'_i \Delta z_i \tag{3.2}$$

where ΔZ is the change in log(productivity), variables with a prime represent values at the final period and upper case letters represent the average of the whole sector, regardless of size. The right-hand side terms denote the between and within effects, respectively.

Each of these effects can be further decomposed to represent three classes of firm's size, corresponding to the categories defined before:

$$\begin{aligned} \Delta Z = & \sum_{i \in C,S} \Delta s_i (z_i - Z) + \sum_{i \in C,S} s'_i \Delta z_i + \\ & \sum_{i \in C,M} \Delta s_i (z_i - Z) + \sum_{i \in C,M} s'_i \Delta z_i + \\ & \sum_{i \in C,L} \Delta s_i (z_i - Z) + \sum_{i \in C,L} s'_i \Delta z_i \end{aligned} \tag{3.3}$$

where S, M and L represents firms that are small, medium and large, respectively. This division helps to elucidate the characteristics that these effects share with firm's size. By making this kind of evaluation, traces that could inevitably be lost by a more sectoral analysis are kept. And it helps to see whether the relevance to productivity comes more from the type of product or by the scale of the business. Of course, any analysis of this kind is not definitive, but it is interesting to see if this promotes another kind of paradigm regarding the relevance of intra-sectoral investigation.

3.5 Results

The results of the decomposition are presented in Figures 17 and 18, for manufacturing and service sectors, respectively. These figures report the cumulated productivity change,

[10] For the derivation in its modern form, please refer to Luna *et al.* (2015), Holm (2010) and Frank (1995).

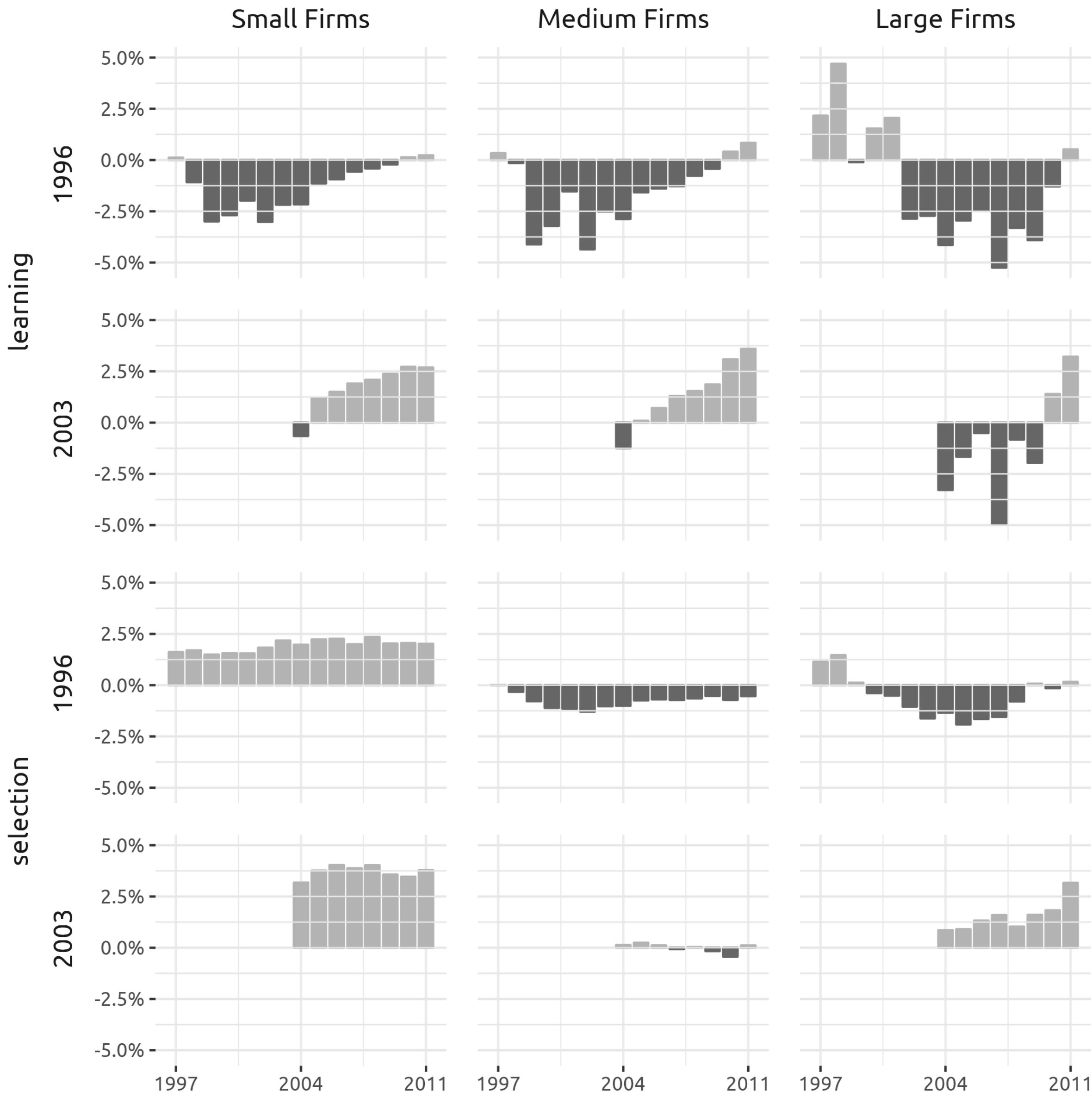

Figure 17 – Manufacturing - Decomposition. Cumulated Results for small, medium and large firms. 1996 and 2003 refers to the base-year of each analysis.

so each base-year is compared against each of the end-years. There is a noticeable difference in the trends of each time period, especially in the within effect of smaller firms of manufacturing. This change in the pattern coincides with the beginning of the commodities cycle.

Also, it is interesting to observe that, for both periods and sectors, the between effect is not meaningful to explain changes in productivity for medium and large firms. This contrasts with the common argument of the efficiency of the market to promote the survival of the fittest, and adds to the international evidence about the small overall contribution of this effect to promote changes in aggregate productivity. As these firms have the largest market-share, it is not surprising that this effect, which is very relevant for

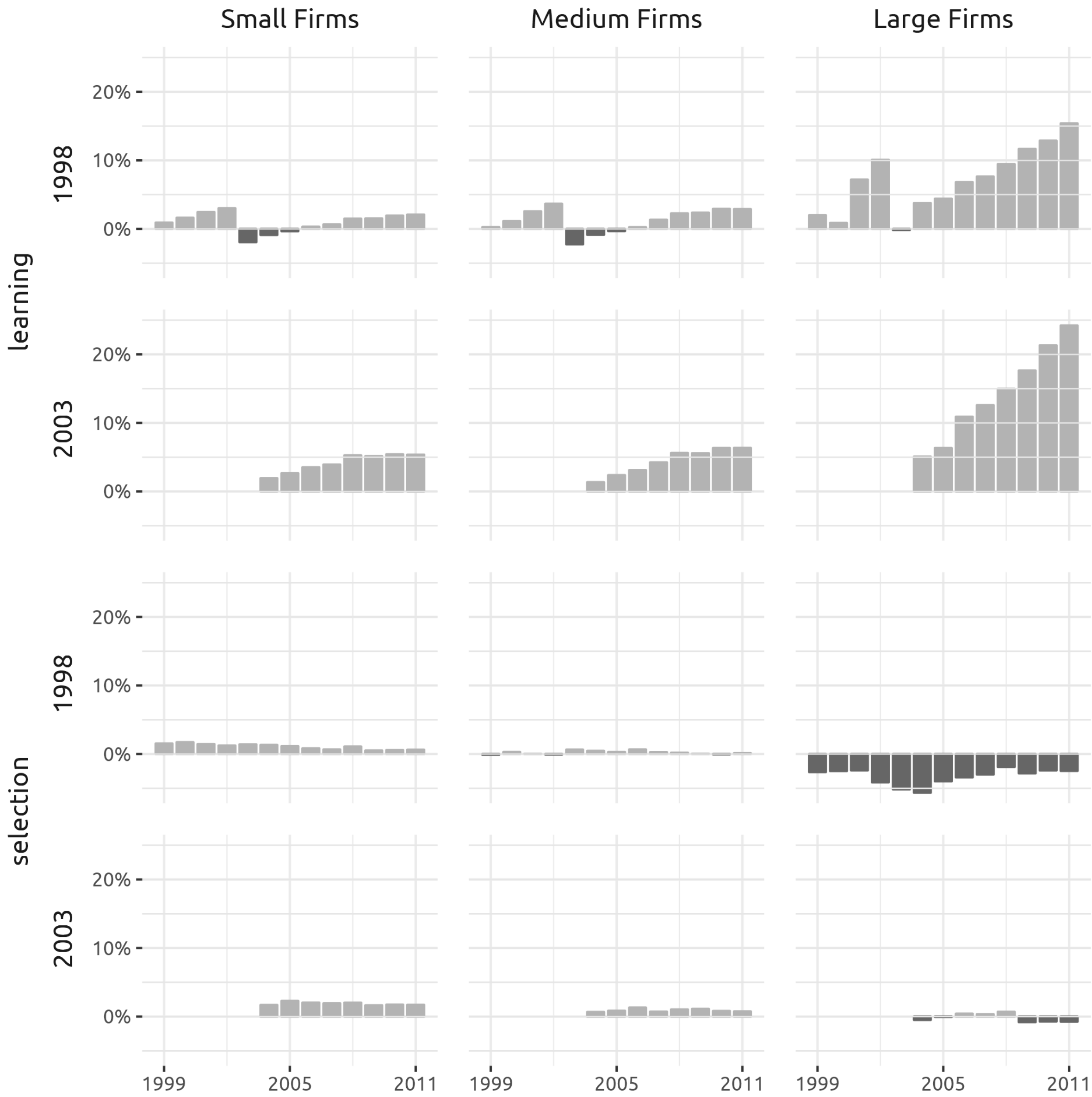

Figure 18 – Services - Decomposition. Cumulated Results for small, medium and large firms. 1998 and 2003 refers to the base-year of each analysis.

small firms, doesn't appear so relevant at the aggregate level.

Our evidence shows that even when base and end-years are far apart, the effect of selection is low when compared to the learning effect. Other works corroborate that this weak effect does not change when multiannual averages are considered (BOTTAZZI *et al.*, 2010), and that the correlation between productivity and growth usually is not statistically significant for lagged periods greater than one (DOSI *et al.*, 2015).

A small caveat is necessary. Besides the low contribution, one has to be very careful about the type of competition that our proxy addresses here. This type of selection has an implicit hypothesis that there is a mechanism that forces the motion of market-shares towards more productive firms without a clear deductive frame. Therefore, how does it happen? Is the scale of the most productive firms more efficient than the others that it allows them to charge less for the product and still have a more productive plant? Or is the quality of their product so superior that they can charge more per unit, thus making their workers more "productive"? There is no easy answer for these questions, and even when using sectoral deflators, there is still a lot of dispersion on prices due to market niches, differences in quality and brand power. Therefore, from what the decomposition allows us to infer, we can observe that the weakness of the selection effect observed in our data can be a consequence of: 1) a low standard deviation on relative productivities (related to the term $z_i - Z$), which as we will show (Tables 9 and 10), is not observed in the data; 2) a low variation of market-shares (related to the term Δs_i), which points to the fact that, regardless of productivity differentials, market-shares are somewhat unchanged over time; and, finally 3) the result of a low correlation between these two metrics, which implies that changes in market size are not related to higher levels of productivity. Both 2) and 3) are expected if this kind of competition defined by productivity differentials is not so important to establish the winners of the market.

So, while considering that the research in the literature in general and in our research in particular, still produces only very rough pictures of the complex processes that affect market selection, particularly due to the lack of more detailed data, our results led us to believe that, as a first insight, other types of mechanisms, like cultural selection[11], may be more relevant to explain how competition works.

Finally, and overall an aspect that the literature has not considered so far, beyond the fact that firms in the same sector may not compete among themselves, some of them may also not produce goods for the final market. The existence of intermediate consumption is especially high in sectors of chemicals, processing of mineral products, food, machinery and vehicles. These firms may then be inserted in a complex network of production, such as the ones depicted in Sturgeon (2002) or in Gereffi *et al.* (2005), having their capacity of setting prices and growing hindered due to monopsony power of leading firms. This type of analysis requires a much more complex representation of the market, which asks for knowledge about the linkages among firms and a careful study about the types of contracts they establish, facts for which in general we lack sufficient data and established methodologies, but that we are actively researching.

In order to enrich the discussion, some complementary descriptive statistics for our main variables are presented in Tables 9 and 10 for manufacturing and for every period analyzed. Tables 11 and 12 show descriptive statistics for the service sector.

First, these evidences drive us to question the significant differences in productivity for different firm's sizes. Why selection by fitness seems to be significant only for the small ones, regardless of the sector? This may be a consequence of their restricted access to credit and finance, but certainly more studies are necessary to address that. Nonetheless,

[11] Cultural selection is based on the same idea that drives evolutionary selection, but change the focus from the price mechanism to other sociological traits, like changes in tastes, fashion and mass culture. It is a form of group selection.

Table 9 – Manufacturing - Statistical Summary. Base-year 1996

Year	Number of firms	Productivity		Employees		Value Added		Net Revenue	
		Mean	Sd	Mean	Sd	Mean	Sd	Mean	Sd
Small Firms									
1997	11,419	64	201	52	22	3.2	7.2	7.6	15.2
1998	11,357	65	166	49	22	3.1	6.0	7.6	15.4
1999	9,971	67	632	51	22	2.8	6.2	7.2	17.4
2000	8,335	63	472	54	22	3.0	5.8	7.9	17.5
2001	7,815	67	502	54	22	3.1	7.3	8.3	21.2
2002	7,147	50	122	54	22	2.7	5.3	7.2	17.5
2003	6,722	58	392	54	22	2.8	5.8	7.7	17.9
2004	6,072	60	382	55	22	2.9	8.2	8.3	41.2
2005	5,742	76	967	54	22	3.2	14.6	9.2	55.8
2006	5,390	70	508	55	22	3.1	8.4	9.1	43.6
2007	4,831	67	345	56	22	3.2	6.2	9.4	45.7
2008	4,593	66	484	56	23	3.1	6.4	8.7	24.3
2009	4,393	63	179	55	22	3.1	4.7	8.3	19.6
2010	4,026	76	514	56	22	3.5	6.4	9.2	29.8
2011	3,865	70	187	56	22	3.5	4.6	8.9	22.7
Medium Firms									
1997	5,375	99	331	211	102	22	53	52	104
1998	4,939	103	312	209	100	23	56	55	104
1999	4,731	89	153	211	101	21	43	50	101
2000	4,582	90	179	211	101	21	64	53	123
2001	4,314	94	166	210	100	21	43	54	117
2002	4,129	85	241	212	100	19	46	49	105
2003	3,969	87	218	213	100	20	45	53	115
2004	3,931	81	235	215	101	18	44	49	105
2005	3,800	84	164	219	104	20	52	53	110
2006	3,695	82	140	219	103	19	52	52	108
2007	3,649	82	153	220	104	19	60	52	109
2008	3,417	82	145	222	104	19	60	53	105
2009	3,281	83	159	221	104	20	67	53	106
2010	3,301	87	160	222	105	21	69	53	102
2011	3,208	86	95	224	105	20	27	54	87
Large Firms									
1997	1,268	147	168	1,491	2,240	279	1,451	619	3,311
1998	1,128	161	198	1,463	2,140	312	1,844	670	3,274
1999	1,074	156	239	1,480	2,175	314	2,310	644	3,449
2000	1,081	155	229	1,497	2,165	341	2,940	692	4,119
2001	1,063	157	282	1,531	2,236	348	2,787	729	4,298
2002	1,074	132	194	1,537	2,197	299	2,247	642	3,446
2003	1,089	137	203	1,555	2,234	330	3,015	714	4,396
2004	1,165	128	199	1,626	2,388	317	2,801	694	4,062
2005	1,138	130	211	1,658	2,406	336	3,132	742	4,534
2006	1,160	130	186	1,654	2,530	333	3,315	726	4,830
2007	1,192	123	171	1,720	2,719	325	3,141	742	4,878
2008	1,151	127	239	1,781	2,955	357	3,676	783	5,682
2009	1,125	119	147	1,826	3,083	340	3,051	770	4,721
2010	1,175	122	160	1,857	3,079	370	3,294	795	5,069
2011	1,177	130	241	1,894	3,240	393	3,553	854	5,490

Source: Our elaboration. Productivity is scaled to BRL 1K, while value added and net revenue are expressed in BRL 1M. Observe that the number of firms may increase above the levels of the base-year, since firms that are considered small in one period can become medium or large firms in subsequent periods, i.e., we opted for a dynamic categorization of each firm.

Table 10 – Manufacturing - Statistical Summary. Base-year 2003

Year	Number of firms	Productivity		Employees		Value Added		Net Revenue	
		Mean	Sd	Mean	Sd	Mean	Sd	Mean	Sd
Small Firms									
2004	16,680	46	246	51	22	2.2	6.6	6.1	27.4
2005	14,983	56	659	52	22	2.4	9.9	6.7	36.1
2006	13,453	53	340	52	22	2.5	6.9	6.8	29.3
2007	11,527	59	387	54	22	2.7	6.4	7.4	31.6
2008	10,683	53	328	54	22	2.6	6.5	7.1	19.4
2009	9,931	54	141	54	22	2.7	4.4	7.1	16.4
2010	8,947	62	360	55	22	3.0	6.0	7.6	22.1
2011	8,531	63	267	55	22	3.0	5.0	7.7	18.4
Medium Firms									
2004	6,343	74	199	203	99	16	38	45	97
2005	6,072	76	151	207	101	17	45	48	101
2006	5,988	76	130	207	101	17	44	48	101
2007	6,029	75	138	208	101	17	50	48	104
2008	5,705	77	139	209	101	17	50	49	96
2009	5,406	79	153	209	100	18	55	48	94
2010	5,475	82	146	210	101	19	56	48	93
2011	5,328	83	141	213	102	19	33	50	90
Large Firms									
2004	1,461	126	198	1,656	2,625	295	2,513	670	3,669
2005	1,446	128	208	1,708	2,763	310	2,790	711	4,069
2006	1,472	126	176	1,724	2,894	306	2,951	697	4,328
2007	1,538	120	162	1,779	3,103	297	2,774	707	4,342
2008	1,503	122	216	1,825	3,337	321	3,227	742	5,024
2009	1,488	119	157	1,812	3,235	301	2,660	719	4,158
2010	1,572	118	152	1,840	3,339	319	2,856	730	4,438
2011	1,571	124	216	1,890	3,459	341	3,084	787	4,809

Source: Our elaboration. Productivity is scaled to BRL 1K, while value added and net revenue are expressed in BRL 1M. Observe that the number of firms may increase above the levels of the base-year, since firms that are considered small in one period can become medium or large firms in subsequent periods, i.e., we opted for a dynamic categorization of each firm.

it was already shown that Brazilian policies on innovation, as the “Lei do Bem”, acted mostly on firms that innovate before its implementation, and from these, more than 80% had more than 500 employees (CALZOLAIO, 2011). This little external support probably creates an environment that is much more harsh and less creative for small enterprises.

Also, the coefficient of variation (not reported) of productivity diminishes in greater classes of firm size, pointing, as we saw in Chapter 2 for some kind of convergence towards a “efficiency frontier”. When controlled by size, this distribution seems to be much less heterogeneous as the unconstrained evidence for other countries (YU *et al.*, 2015a; DOSI *et al.*, 2015). This could mean that either 1) size measured by number of employees is as important as a sectoral analysis to ascertain productivity deviation and that 2) the capital intensity of each enterprise, at least for Manufacturing, is intimately related to its size as measured here.

Table 11 – Services - Statistical Summary. Base-year 1998

Year	Number of firms	Productivity		Employees		Value Added		Net Revenue	
		Mean	Sd	Mean	Sd	Mean	Sd	Mean	Sd
Small Firms									
1999	13,408	57	197	37	21	1.9	5.6	2.8	7.5
2000	12,060	68	840	38	21	2.0	6.2	3.0	8.8
2001	10,820	64	386	39	21	2.1	5.4	3.2	7.9
2002	9,937	62	218	39	21	2.1	6.3	3.1	9.0
2003	9,153	59	225	39	21	2.1	5.3	3.0	7.4
2004	8,871	71	523	38	21	2.2	5.9	3.3	8.2
2005	8,363	71	553	39	22	2.1	5.3	3.3	7.9
2006	7,819	97	2,253	39	22	2.2	5.5	3.4	8.0
2007	6,992	177	7,533	40	22	2.4	9.3	3.7	11.6
2008	6,623	79	561	41	21	2.6	6.2	3.8	9.0
2009	6,301	101	2,554	41	21	2.6	7.1	3.8	10.0
2010	6,077	95	1,429	42	21	2.8	8.1	4.1	11.2
2011	5,900	94	776	42	21	2.9	9.8	4.2	14.3
Medium Firms									
1999	3,041	50	78	215	102	10	17	15	27
2000	2,919	52	92	219	106	11	19	17	31
2001	2,787	51	65	222	108	11	18	17	30
2002	2,606	54	71	222	107	11	16	17	28
2003	2,447	57	76	222	108	12	19	18	31
2004	2,399	58	75	223	106	13	20	19	34
2005	2,317	60	73	224	107	13	18	20	31
2006	2,209	60	69	227	107	13	18	21	30
2007	2,136	63	73	229	110	14	17	21	29
2008	2,045	66	72	231	110	15	18	23	31
2009	2,028	69	82	229	110	15	20	23	31
2010	1,998	71	79	230	109	16	22	24	34
2011	1,955	71	77	231	109	16	22	24	31
Large Firms									
1999	950	53	142	1,382	3,025	81	308	125	554
2000	960	47	76	1,390	2,981	75	293	119	595
2001	942	45	67	1,427	3,284	79	393	129	812
2002	957	48	87	1,449	3,488	80	429	133	872
2003	895	49	104	1,506	3,860	84	455	133	858
2004	956	50	98	1,536	3,952	86	474	134	853
2005	960	49	94	1,493	1,861	78	382	124	783
2006	937	52	93	1,551	2,057	85	383	139	806
2007	936	54	99	1,632	2,258	96	406	154	834
2008	921	54	87	1,632	2,052	90	340	144	769
2009	954	57	105	1,721	4,010	105	415	170	882
2010	973	57	93	1,748	3,972	108	419	175	854
2011	1,006	59	87	1,819	4,315	112	418	180	823

Source: Our elaboration. Productivity is scaled to BRL 1K, while value added and net revenue are expressed in BRL 1M. Observe that the number of firms may increase above the levels of the base-year, since firms that are considered small in one period can become medium or large firms in subsequent periods, i.e., we opted for a dynamic categorization of each firm.

Table 12 – Services - Statistical Summary. Base-year 2003

Year	Number of firms	Productivity		Employees		Value Added		Net Revenue	
		Mean	Sd	Mean	Sd	Mean	Sd	Mean	Sd
Small Firms									
2004	21,391	58	188	36	21	1.8	4.8	2.7	7.4
2005	18,925	59	218	38	21	1.9	5.2	2.9	8.2
2006	17,148	77	1,550	38	21	2.0	5.3	3.0	8.6
2007	14,651	112	5,112	40	21	2.2	7.8	3.3	10.3
2008	13,700	82	830	40	21	2.4	6.4	3.5	10.2
2009	12,802	87	1,807	41	21	2.5	7.9	3.6	11.9
2010	12,107	87	1,043	41	21	2.7	7.2	3.8	11.7
2011	11,464	91	806	41	21	2.8	8.4	4.0	14.1
Medium Firms									
2004	4,159	59	136	213	103	12	31	18	46
2005	3,977	59	124	215	104	13	31	19	49
2006	3,736	59	96	219	106	13	23	19	36
2007	3,596	64	115	221	107	14	24	21	41
2008	3,448	66	96	223	107	15	25	22	44
2009	3,360	69	101	222	107	15	24	22	36
2010	3,359	71	108	223	107	16	26	24	42
2011	3,271	72	104	223	107	16	26	24	41
Large Firms									
2004	1,267	61	144	1,542	3,874	100	480	155	832
2005	1,300	57	124	1,539	2,731	92	404	144	760
2006	1,282	55	110	1,594	2,915	91	371	147	735
2007	1,299	56	113	1,683	3,327	100	425	160	835
2008	1,312	58	116	1,710	3,553	101	405	162	817
2009	1,362	60	136	1,789	4,584	109	437	176	886
2010	1,384	62	163	1,855	4,773	115	453	183	881
2011	1,435	63	134	1,927	5,068	120	458	190	887

Source: Our elaboration. Productivity is scaled to BRL 1K, while value added and net revenue are expressed in BRL 1M. Observe that the number of firms may increase above the levels of the base-year, since firms that are considered small in one period can become medium or large firms in subsequent periods, i.e., we opted for a dynamic categorization of each firm.

Another interesting fact is the evidence of scale gains for Manufacturing, which does not occur for Services. In this last sector, scale gains without affecting quality are inherently difficult to be made, and measures of productivity are extremely difficult to be validated (BAUMOL, 1967; BAUMOL *et al.*, 2012). The last point, however, does not downgrades the results by itself, but claims precaution, as there are "shadow" quality improvements that disappear when looking at prices only[12].

A more detained vision to market-share is also necessary. The greater share of market is due to large firms, which for manufacturing also happen to be the most productive ones. This is an important insight because the lack of controls for size can mask what

[12] These "shadow" improvements are related to changes in the nature of the service that are not necessarily translated in prices, such as the impacts of the IT Revolution, which Solow remarkably said that "could be saw everywhere but in the productivity statistics" (SOLOW, 1987).

happens at the sectoral level for smaller enterprises, and may lead to a biased view of the process of productivity change.

This is fundamental for public policies, since without a clear depiction of the process of selection, economic liberalization could be obliviously implemented due to an apparent "lack" of competition, which would be particularly onerous on small firms. Unfortunately, to our knowledge, few other studies controlled for size when doing decomposition analysis (DOSI *et al.*, 2015).

A significant distinction appears when one analyses the within effect. While it is hard to point a precise trend in the results, certain aspects deserve to be mentioned. First, it is interesting to notice that for the service sector there is much more consistency among the different sizes than in manufacturing, with the within effect being positive in a significant part of both periods for all sizes.

In the industrial sector, there seems to be more of an inverse movement in the learning effect observed among small and medium firms versus the large ones, with most of the years showing that the internal movements act distinctively for these two groups. But why? It is hard to find a convincing explanation for this without more data. It does not seem to be related to the investment cycle or downsizing, as the large firms class expanded the average number of employees consistently in both periods, even if more fiercely in the second one. Also, it appears to be greatly influenced by the stage of the economic cycle, contributing negatively in the first period and positively in the second for small and medium firms.

This poses the important question of how much of real, physical productivity is measured when making decomposition studies with monetary productivity. Are these differences consequence of investments in capital and technology or only changes in mark-up and idle capacity due to a higher or lower demand? Well, the evidence in this regard is more dubious, especially because of the lack of data from individual firms' investments. But the relationship between net revenue and value added points to a considerable effect of sales in promoting the within effect, and thus, in the aggregate productivity change.

This supports the idea that firms do have a non-negligible idle capacity and that productivity itself may be highly pro-cyclical and demand-dependant, at least for manufacturing, which would be a sort of micro Kaldor-Verdoorn law. That is because, if firms need a change on market size to expand or contract their productivity, then the productivity change, and the within effect increase, particularly, are not due to an internal transformation, but to the cyclical economic activity. In other words, a fixed mark-up expressed by a high correlation between the net revenue and value added, while keeping the size as measured by employees relatively fixed, would provide evidence for the idea of unaltered productive structure, which Tables 9 and 10 supports for the average values in each class.

This, of course, would not be true if the investments promoted higher sales that were accompanied by a decreasing mark-up[13], or if the better quality products that were made through investments do not acquire a mark-up differential, but are instead passed as a consumer benefit[14]. In this case, these quality gains would be invisible when using monetary productivity. These are valid hypothesis that any study should take care when contrasting monetary and physical productivity.

Another evidence for the idle capacity hypothesis is the importance of the within effect for the smaller firms, which are supposedly less prone and capable to make the required investments in innovation and research. If these variations in productivity are not consequence of R&D, what generates them? The fact that they are consistently negative for these firms in the first period also points to an increase in idle capacity.

Yet, the evidence in this respect is far from conclusive and we plan to address this point more profoundly in the future. Other databases, such as the PIM-PF[15] gives us some leads regarding the changes in physical production for each industry.

There is no database to our knowledge, however, that presents microdata regarding the idle capacity of manufacturing firms, and to date, it is not possible to estimate them without the heroic hypotheses of aggregate production functions. So, it is very hard to disentangle, even when measuring physical input-outputs, the gains in productivity due to investments against the ones caused by changes in idle capacity. This, of course, affects not only this study, but the literature in general.

3.6 Conclusion

Our main results support the presence of a low between effect in the industrial dynamics as found in the international literature as well as the relevance of the idiosyncratic internal behavior to promote aggregate productivity change. They also point to the importance of size as a control for capital intensity, as showed by the decreasing relative standard deviation found for productivity in the higher classes of firm sizes, especially when compared to the other selected metrics. The fact that this deviation diminishes as firms get bigger for manufacturing also gives us some support to the idea of technological frontiers. On the other hand, in the service sector, the lack of scale gains shows that it is very hard to promote efficiency in a sector that depends fundamentally on human hours of work to produce its *goods*, besides the difficulties of measurement already mentioned and the "shadow" improvements in quality as well. The great market-share of larger firms, likewise, shows the vulnerabilities of decomposition studies that do not make distinctions based on number of employees to represent the whole landscape of the market, as smaller

[13] An example would be sectors that are being made obsolete by new technologies.

[14] In other words, there is improvement in the final product but the enterprise is not able to charge more for it. Investments in this case are made just for the firm to hold market-share and not being "eaten" by the competition.

[15] This database reports the volume of production of goods for different manufacturing sectors.

firms seem to be much more affected by competition than the larger ones. Lastly, there is some indication that the within effect can be a representation of the economic cycle, and highly idle-capacity dependent, but more studies are necessary to address that.

Further improvements on this research and promising venues are related to the study of the contrast between physical and monetary productivity decompositions, the exploration and consequences of the existence of intermediary consumption, such as the existence of networks of firms, and with them, hierarchies, and disaggregation of these analyses both by size and sub-sectors of manufacturing and services.

Finally, the relationship between the economic cycle and the within effect also needs to be tested by a direct approach using investment data and productivity change.

Conclusion

This dissertation aimed to make a broad characterization of Brazilian Manufacturing, and, in a smaller part, the Service sector, regarding the most important stylized facts found in Industrial Organization. Chapter 1 brought a small revision of the literature, while Chapter 2 and Chapter 3 contributed with novel results.

Particularly, in Chapter 1 we presented the evolution of the research regarding patterns in industrial micro data, such as the Gibrat Law, the lognormal and Pareto shape of firm size distributions and the Laplacian shape of firm rates, among others.

In Chapter 2, we presented statistical exercises regarding Brazilian Manufacturing. Our contributions provided evidence of a ubiquitous heterogeneity in the most important metrics of size, growth and productivity. There is also compelling evidence regarding both the Pareto and lognormal shape of firm size distributions, which seem robust to disaggregation and persistent in time, at least as a first approximation. Firm rates distributions have a symmetrical shape, well described by an AEP distribution, with most tails at least Laplacian. Finally, productivity appears to have an asymmetrical shape, with some evidence of an "efficiency frontier" that limits the performance of the market leaders, while the left side of the distribution is mostly unconstrained and assumes fatter tails.

In Chapter 3, we investigated the Brazilian market dynamics using a decomposition exercise of productivity change. This exercise produced evidence of a low between effect, as found in the international literature, but which affects firms distinctly as categorized by classes of size. Specifically, smaller firms appear to be more affected by our proxies of competition, while for bigger firms, competition doesn't appear to "bite" as much as previously thought. So, studies regarding decomposition of productivity change may benefit to incorporate size categorizations, as at least for Brazil, size does matter. Also, regarding the firm-specific internal variation, learning appears to be highly correlated to the economic cycle, and represents most of productivity variation.

Overall, this work presents evidence contrary to the hypothesis of an optimal size of firms or the existence of a representative agent. Firm size distributions are very skewed and with a wide dispersion, even acquiring bimodalities and non-smooth shapes. If well-behaved *u-shaped* cost curves would be a meaningful representation of the markets, one would expect a more defined trend to convergence for an optimal size, at least inside the sectors.

Among the hypotheses for such a dissimilar performance we listed factors such as differences in firm capabilities, scales of operation and access to better prices through suppliers. Beyond those, market niches and brand power may create differential of earnings that would not be mitigated even if firms shared the same costs or technology (STURGEON, 2002).

This heterogeneity may as well be a emerging property of capitalist societies, and may serve functions that we are still unaware, but that may be comprehended using tools from network theory. Hierarchies can constitute an easier way to transmite signaling information, helping to organize markets (HAYEK, 1945; KRUGMAN, 1996b), and at same time be more robust to random shocks, while also reducing the distance between agents (BARABASI, 2016). At the same time, they are more fragile against meltdowns of important players, and “too big to fail” (NURISSO; PRESCOTT, 2017) is now a household term. Yet, more work is necessary regarding the goodness of Pareto fits against other distributions.

Regarding productivity distributions, beyond the concept of an “efficiency frontier”, the fat left tail may reflect not a low physical productivity in itself, but a low capacity of these firms to capture market earnings, and their adverse positioning in the production network (STURGEON, 2002; GEREFFI *et al.*, 2005). This would be especially strong if they are producing for intermediate consumption, which may make them captives of the monopsony power from the leading firms.

Finally, the growth rate distributions show the presence of some kind of short-run correlation among the events that produced growth, which were effectively modeled in the Simon’s tradition of “islands of opportunity” (BOTTAZZI; SECCHI, 2006a; IJIRI; SIMON, 1977). At the same time, these distributions contrast with notions from innovation theory, such as capabilities and technological trajectories. These two concepts, together, would make one expect for fairly smooth periods of incremental perfecting followed by large jumps of rapid growth due to change of paradigms.

The characteristic Laplacian shape for firm growth rates contradicts this view and suggests that there are important middle steps between what configures learning in the sense of technological advancement and organizational management and what in fact generates financial returns. So, these theories must be adapted to faithfully incorporate the kind of short-term competition and the network nature of markets, such as to define who is more probably to take the gains of innovation: costumers, leaders or innovators. (GEREFFI *et al.*, 2005).

Also, the study of market selection requires a deeper intuition of the links between productivity change and growth, and their relationship with size. At this moment, this kind of analysis can produce only very rough results, which need the advancements that competition at the product and regional level would give. Our own proxy of productivity must be improved to capture changes at the physical level, and correctly filter the impacts of the economic cycle.

Bibliography

ACEMOGLU, D.; AKCIGIT, U.; BLOOM, N.; KERR, W. R. Innovation, reallocation and growth. NBER working paper 18993. National Bureau of Economic Research. 2013.

AMARAL, L.; BULDYREV, S.; HAVLIN, S.; MAASS, P.; SALINGER, M.; STANLEY, H.; STANLEY, M. Scaling behavior in economics: The problem of quantifying company growth. *Physica A: Statistical Mechanics and its Applications*, v. 244, n. 1, p. 1–24, 1997. ISSN 0378-4371. Available at: <http://www.sciencedirect.com/science/article/pii/S0378437197003014>.

ANDERSEN, E. S. Population thinking, Price's equation and the analysis of economic evolution. *Evolutionary and Institutional Economics Review*, v. 1, n. 1, p. 127–148, 2004.

ARROW, K. The economic implications of learning by doing. *Review of Economic Studies*, v. 29, n. 3, p. 155–173, 1962. Available at: <https://EconPapers.repec.org/RePEc:oup:restud:v:29:y:1962:i:3:p:155-173.>

ATKINSON, A.; PIKETTY, T. *Top Incomes over the Twentieth Century: A Contrast between European and English-Speaking Countries*. [S.l.: s.n.], 2007. ISBN 0199286884,9780199286881,9781429493000.

AUDRETSCH, D.; KLOMP, L.; SANTARELLI, E.; THURIK, R. Gibrat's law: Are the services different? *Review of Industrial Organization*, v. 24, n. 3, p. 301–324, 2004. Available at: <http://EconPapers.repec.org/RePEc:kap:revind:v:24:y:2004:i:3:p:301-324>.

AUDRETSCH, D.; SANTARELLI, E.; VIVARELLI, M. Start-up size and industrial dynamics: some evidence from Italian manufacturing. *International Journal of Industrial Organization*, v. 17, n. 7, p. 965–983, 1999. Available at: <http://EconPapers.repec.org/RePEc:eee:indorg:v:17:y:1999:i:7:p:965-983>.

AUGUIE, B. *gridExtra: Miscellaneous Functions for "Grid" Graphics*. [S.l.], 2017. R package version 2.3. Available at: <https://CRAN.R-project.org/package=gridExtra>.

AXTELL, R. L. Zipf distribution of U.S. firm sizes. *Science*, American Association for the Advancement of Science, v. 293, n. 5536, p. 1818–1820, 2001. ISSN 0036-8075. Available at: <http://science.sciencemag.org/content/293/5536/1818>.

BACHE, S. M.; WICKHAM, H. *magrittr: A Forward-Pipe Operator for R*. [S.l.], 2014. R package version 1.5. Available at: <https://CRAN.R-project.org/package=magrittr>.

BAILY, M. N.; BARTELSMAN, E. J.; HALTIWANGER, J. Downsizing and productivity growth: myth or reality? *Small Business Economics*, v. 8, p. 259–278, 1996.

BAILY, M. N.; HULTEN, C.; CAMPBELL, D. Productivity dynamics in manufacturing establishments. *Brookings Papers on Economic Activity: Microeconomics*, v. 4, p. 187–249, 1992.

BALDWIN, J. R.; GU, W. Plant turnover and productivity growth in Canadian manufacturing. *Industrial and Corporate Change*, v. 15, p. 417–465, 2006.

BARABASI, A.-L. *Network Science*. United Kingdom: Cambridge University Press, 2016. ISBN 978-1107076266.

BARTELSMAN, E. J.; DHRYMES, P. J. Productivity dynamics: U.S. manufacturing plants. *Journal of Productivity Analysis*, v. 9, n. 1, p. 5–34, 1998.

BARTELSMAN, E. J.; DOMS, M. Understanding productivity: lessons from longitudinal microdata. *Journal of Economic Literature*, v. 38, p. 569–594, 2000.

BAUMOL, W.; FERRANTI, D. D.; MALACH, M.; PABLOS-MÉNDEZ, A.; TABISH, H.; WU, L. *The Cost Disease: Why Computers Get Cheaper and Health Care Doesn't*. New Haven & London: Yale University Press, 2012.

BAUMOL, W. J. Macroeconomics of unbalanced growth: The anatomy of urban crisis. *The American Economic Review*, v. 57, n. 3, p. 415–426, 1967.

BEE, M.; RICCABONI, M.; SCHIAVO, S. The size distribution of US cities: Not Pareto, even in the tail. *Economics Letters*, v. 120, n. 2, p. 232–237, 2013. ISSN 0165-1765. Available at: <http://www.sciencedirect.com/science/article/pii/S0165176513002139>.

BONELLI, R.; BACHA, . L. In: _____. *Desenvolvimento econômico: uma perspectiva Brasileira*. [S.l.]: Elsevier, 2013. ch. Crescimento brasileiro revisitado, p. 236–271.

BOTTAZZI, G.; CEFIS, E.; DOSI, G.; SECCHI, A. Invariances and diversities in the patterns of industrial evolution: Some evidence from Italian manufacturing industries. *Small Business Economics*, v. 29, p. 137–159, 2007.

BOTTAZZI, G.; DOSI, G.; JACOBY, N.; SECCHIY, A.; TAMAGNI, F. Corporate performances and market selection: some comparative evidence. *Industrial and Corporate Change*, v. 19, n. 6, p. 1953–1996, 2010.

BOTTAZZI, G.; DOSI, G.; ROCCHETTI, G. Modes of knowledge accumulation, entry regimes and patterns of industrial evolution. *Industrial and Corporate Change*, v. 10, n. 3, p. 609–638, 2001.

BOTTAZZI, G.; PIRINO, D.; TAMAGNI, F. Zipf law and the firm size distribution: a critical discussion of popular estimators. *Journal of Evolutionary Economics*, v. 25, n. 3, p. 585–610, Jul 2015. ISSN 1432-1386. Available at: <https://doi.org/10.1007/s00191-015-0395-7>.

BOTTAZZI, G.; SECCHI, A. Common properties and sectoral specificities in the dynamics of U.S. manufacturing companies. *Review of Industrial Organization*, v. 23, p. 217–232, 2003.

BOTTAZZI, G.; SECCHI, A. Growth and diversification patterns of the worldwide pharmaceutical industry. *Review of Industrial Organization*, v. 26, p. 195–216, 2005.

BOTTAZZI, G.; SECCHI, A. Explaining the distribution of firm growth rates. *The RAND Journal of Economics*, Blackwell Publishing Ltd, v. 37, n. 2, p. 235–256, 2006. ISSN 1756-2171. Available at: <http://dx.doi.org/10.1111/j.1756-2171.2006.tb00014.x>.

BOTTAZZI, G.; SECCHI, A. Gibrat's law and diversification. *Industrial and Corporate Change*, v. 15, n. 5, p. 847–875, 2006. Available at: <+http://dx.doi.org/10.1093/icc/dtl019>.

BOTTAZZI, G.; SECCHI, A. A new class of asymmetric exponential power densities with applications to economics and finance. *Industrial and Corporate Change*, v. 20, n. 4, p. 991–1030, 2011. Available at: <+http://dx.doi.org/10.1093/icc/dtr036>.

BULDYREV, S.; AMARAL, L.; HAVLIN, S.; LESCHHORN, H.; MAASS, P.; SALINGER, M.; STANLEY, H.; STANLEY, R. Scaling behavior in economics: Ii. modeling of company growth. v. 7, 02 1997.

CABRAL, L. M. B.; MATA, J. On the evolution of the firm size distribution: Facts and theory. *American Economic Review*, v. 93, n. 4, p. 1075–1090, September 2003. Available at: <http://www.aeaweb.org/articles?id=10.1257/000282803769206205>.

CALZOLAIO, A. *Política Fiscal de Incentivo à Inovação no Brasil: Análise do Desempenho Inovativo das Empresas que Usufruíram Benefícios da Lei nº 11.196/05 (Lei do Bem).* Dissertation (Master's Degree) — Universidade Federal do Rio Grande do Sul, 2011.

CANTNER, U.; KRUGER, J. Micro-heterogeneity and aggregate productivity development in the German manufacturing sector: Results from a decomposition exercise. *Journal of Evolutionary Economics*, v. 18, n. 2, p. 119–133, 2008.

CATELA, E.; CIMOLI, M.; PORCILE, G. Productivity and structural heterogeneity in the Brazilian manufacturing sector: Trends and determinants. *Oxford Development Studies*, v. 43, n. 2, p. 232–252, 2015.

CEPAL. *La Hora de la Igualdad: Brechas por cerrar, caminos por abrir.* Santiago: Naciones Unidas, 2010.

CHAMPERNOWNE, D. G. A model of income distribution. *The Economic Journal*, [Royal Economic Society, Wiley], v. 63, n. 250, p. 318–351, 1953. ISSN 00130133, 14680297. Available at: <http://www.jstor.org/stable/2227127>.

CHANG, W. *extrafont: Tools for using fonts*. [S.l.], 2014. R package version 0.17. Available at: <https://CRAN.R-project.org/package=extrafont>.

CIMOLI, M.; PORCILE, G. Tecnología, heterogeneidad y crescimiento. *ECLAC - Serie Desarrollo Productivo*, v. 194, p. 1–38, Sep 2013. ISSN 1020-5179. Available at: <http://repositorio.cepal.org/handle/11362/4592>.

CLAUSET, A.; SHALIZI, C. R.; NEWMAN, M. E. J. Power-law distributions in empirical data. *SIAM Review*, 2009.

CROISSANT, Y.; MILLO, G. Panel data econometrics in R: The plm package. *Journal of Statistical Software*, v. 27, n. 2, 2008. Available at: <http://www.jstatsoft.org/v27/i02/>.

CROSATO, L.; GANUGI, P. Statistical regularity of firm size distribution: the Pareto IV and truncated Yule for Italian SCI manufacturing. *Statistical Methods and Applications*, v. 16, n. 1, p. 85–115, Jun 2007. ISSN 1613-981X. Available at: <https://doi.org/10.1007/s10260-006-0023-7>.

DISNEY, R.; HASKEL, J.; HEDEN, Y. Entry, exit and establishment survival in UK manufacturing. *Journal of Industrial Economics*, v. 51, p. 91–112, 2003.

DISNEY, R.; HASKEL, J.; HEDEN, Y. Restructuring and productivity growth in UK manufacturing. *The Economic Journal*, v. 113, n. 489 (Jul., 2003), p. 666–694, 2003.

DOSI, G. Technological paradigms and technological trajectories: A suggested interpretation of the determinants and directions of technical change. *Research Policy*, v. 11, n. 3, p. 147–162, 1982. Available at: <https://EconPapers.repec.org/RePEc:eee:respol:v:11:y:1982:i:3:p:147-162>.

DOSI, G. Statistical regularities in the evolution of industries. a guide through some evidence and challenges for the theory. LEM Working Papers n 17/05. 2005.

DOSI, G.; FAGIOLO, G.; ROVENTINI, A. Schumpeter meeting Keynes: A policy-friendly model of endogenous growth and business cycles. *Journal of Economic Dynamics and Control*, v. 34, p. 1748–1767, 2010.

DOSI, G.; GAMBARDELLA, A.; GRAZZI, M.; ORSENIGO, L. Technological revolutions and the evolution of industrial structures: Assessing the impact of new technologies upon the size and boundaries of firms. *Capitalism and Society*, v. 3, n. 1, 2008.

DOSI, G.; GRAZZI, M.; TOMASI, C.; ZELI, A. Turbulence underneath the big calm? the micro-evidence behind Italian productivity dynamics. *Small Business Economics*, v. 39, n. 4, p. 1043–1067, 2012. ISSN 1573-0913. Available at: <http://dx.doi.org/10.1007/s11187-011-9326-7>.

DOSI, G.; MARSILI, O.; ORSENIGO, L.; SALVATORE, R. Learning, market selection and evolution of industrial structures. *Small Business Economics*, v. 7, p. 411–436, 1995.

DOSI, G.; MOSCHELLA, D.; PUGLIESE, E.; TAMAGNI, F. Productivity, market selection, and corporate growth:comparative evidence across US and Europe. *Small Business Economics*, v. 45, n. 3, p. 643–672, 2015.

DOSI, G.; NELSON, R. In: ______. *Handbook of the Economics of Innovation,*. New York: North Holland., 2010. v. 1, ch. Technical Change And Industrial Dynamics As Evolutionary Processes.

DOSI, G.; PEREIRA, M. C.; VIRGILLITO, M. E. The footprint of evolutionary processes of learning and selection upon the statistical properties of industrial dynamics. *Industrial and Corporate Change*, v. 26, n. 2, p. 187, 2017. Available at: <+http://dx.doi.org/10.1093/icc/dtw044>.

DOWLE, M.; SRINIVASAN, A. *data.table: Extension of ‘data.frame‘*. [S.l.], 2017. R package version 1.10.4-3. Available at: <https://CRAN.R-project.org/package=data.table>.

DUNNE, T.; ROBERTS, M.; SAMUELSON, L. Patterns of firm entry and exit in U.S. manufacturing industries. *The RAND Journal of Economics*, [RAND Corporation, Wiley], v. 19, n. 4, p. 495–515, 1988. ISSN 07416261. Available at: <http://www.jstor.org/stable/2555454>.

DUNNE, T.; ROBERTS, M. J.; SAMUELSON, L. The growth and failure of u. s. manufacturing plants. *The Quarterly Journal of Economics*, v. 104, n. 4, p. 671–698, 1989. Available at: <http://EconPapers.repec.org/RePEc:oup:qjecon:v:104:y:1989:i:4:p:671-698.>

ERICSON, R.; PAKES, A. Markov-perfect industry dynamics: a framework for empirical work. *Review of Economic Studies*, v. 62, p. 53–82, 1995.

ESTEVES, L. In: _____. *Produtividade no Brasil: desempenho e determinantes – Vol. 2 - Determinantes*. Brasília: ABD, 2015. ch. Economias de Escala, Economias de Escopo e Eficiência Produtiva na Indústria Brasileira de Transformação.

EVANS, D. The relationship between firm growth, size, and age: Estimates for 100 manufacturing industries. *The Journal of Industrial Economics*, Wiley, v. 35, n. 4, p. 567–581, 1987. ISSN 00221821, 14676451. Available at: <http://www.jstor.org/stable/2098588>.

EVANS, D. Tests of alternative theories of firm growth. *Journal of Political Economy*, University of Chicago Press, v. 95, n. 4, p. 657–674, 1987. ISSN 00223808, 1537534X. Available at: <http://www.jstor.org/stable/1833253>.

FAN, J.; MARRON, J. S. Fast implementations of nonparametric curve estimators. *Journal of Computational and Graphical Statistics*, v. 3, n. 1, p. 35–56, 1994. Available at: <http://www.tandfonline.com/doi/abs/10.1080/10618600.1994.10474629>.

FELIPE, J.; MCCOMBIE, J. *The Aggregate Production Function and the Measurement of Technical Change: Not Even Wrong*. Edward Elgar Publishing, 2013. ISBN 9781782549680. Available at: <https://books.google.com.br/books?id=BevDAQAAQBAJ>.

FERREIRA, P.; JR., R. E.; GOMES, V. Produtividade agregada brasileira (1970-2000): declínio robusto e fraca recuperação. *Estudos Econômicos (São Paulo)*, v. 38, n. 1, p. 31–53, 2008.

FISHER, F. M. Aggregate production functions - a pervasive, but unpersuasive, fairytale. *Eastern Economic Journal*, Palgrave Macmillan Journals, v. 31, n. 3, p. 489–491, 2005. ISSN 00945056, 19394632. Available at: <http://www.jstor.org/stable/40326426>.

FISHER, R. A. *The Genetical Theory of Natural Selection*. Oxford: Clarendon, 1930.

FOSTER, L.; HALTIWANGER, J.; KRIZAN, C. J. In: _____. *New Developments in Productivity Analysis*. Chicago: University of Chicago Press, 2001. ch. Aggregate productivity growth: lessons from microeconomic evidence, p. 303–372.

FRANK, S. A. George Price's contribution to evolutionary genetics. *Journal of Theoretical Biology*, v. 175, n. 3, p. 373–388, 1995.

GABAIX, X. Zipf's law for cities: An explanation. *The Quarterly Journal of Economics*, v. 114, n. 3, p. 739–767, 1999. Available at: <+http://dx.doi.org/10.1162/003355399556133>.

GABAIX, X. Power laws in economics and finance. *Annual Review of Economics*, v. 1, n. 1, p. 255–294, 2009. Available at: <https://doi.org/10.1146/annurev.economics.050708.142940>.

GALA, P.; CAMARGO, J.; MAGACHO, G. The resource curse reloaded: revisiting the Dutch disease with economic complexity analysis. 10 2017. FGV EESP - Textos para Discussão / Working Paper Series - TD 448.

GEREFFI, G.; HUMPHREY, J.; STURGEON, T. The governance of global value chains. *Review of International Political Economy*, Routledge, v. 12, n. 1, p. 78–104, 2005. Available at: <http://dx.doi.org/10.1080/09692290500049805>.

GEROSKI, P. A. What do we know about entry? *International Journal of Industrial Organization*, v. 13, n. 4, p. 421–440, 1995. ISSN 0167-7187. The Post-Entry Performance of Firms. Available at: <http://www.sciencedirect.com/science/article/pii/016771879500498X>.

GEROSKI, P. A.; MACHIN, S. Innovation, profitability and growth over the business cycle. *Empirica*, v. 20, p. 35–50, 1993. Available at: <https://doi.org/10.1007/BF01384139>.

GIBRAT, R. *Les Inégalités économiques*. Paris: Librairie du Recuil Sirey, France, 1931.

GRILICHES, Z.; MAIRESSE, J. In: ______. *Econometrics and Economic Theory in the Twentieth Century: the Ragner Frisch Centennial Symposium*. Cambridge: Cambridge University Press., 1997. ch. Production function: the search for identification.

GRILICHES, Z.; REGEV, H. Firm productivity in Israeli industry 1979–1988. *Journal of Econometrics*, v. 65, p. 175–203, 1995.

HALL, B. H. The relationship between firm size and firm growth in the US manufacturing sector. *The Journal of Industrial Economics*, Wiley, v. 35, n. 4, p. 583–606, 1987. ISSN 00221821, 14676451. Available at: <http://www.jstor.org/stable/2098589>.

HALL, P.; WAND, M. On the accuracy of binned kernel density estimators. *J. Multivar. Anal.*, Academic Press, Inc., Orlando, FL, USA, v. 56, n. 2, p. 165–184, feb. 1996. ISSN 0047-259X. Available at: <http://dx.doi.org/10.1006/jmva.1996.0009>.

HART, P. E.; PRAIS, S. J. The analysis of business concentration: A statistical approach. *Journal of the Royal Statistical Society. Series A (General)*, v. 119, p. 150–191, 1956.

HARTMANN, D.; GUEVARA, M. R.; JARA-FIGUEROA, C.; ARISTARáN, M.; HIDALGO, C. A. Linking economic complexity, institutions, and income inequality. *World Development*, v. 93, p. 75–93, 2017. ISSN 0305-750X. Available at: <http://www.sciencedirect.com/science/article/pii/S0305750X15309876>.

HAUSMANN, R.; HIDALGO, C. *The Atlas of Economic Complexity: Mapping Paths to Prosperity*. 1. ed. [S.l.]: MIT Press Books, The MIT Press, 2014. v. 1. ISBN 9780262317726.

HAYEK, F. A. The use of knowledge in society. *The American Economic Review*, American Economic Association, v. 35, n. 4, p. 519–530, 1945. ISSN 00028282. Available at: <http://www.jstor.org/stable/1809376>.

HENRY, L.; WICKHAM, H. *rlang: Functions for Base Types and Core R and 'Tidyverse' Features*. [S.l.], 2017. R package version 0.1.2. Available at: <https://CRAN.R-project.org/package=rlang>.

HILL, B. A simple general approach to inference about the tail of a distribution. *The Annals of Statistics*, v. 3, p. 1163–1174, 1975.

HODGSON, G.; KNUDSEN, T. The firm as an interactor: firms as vehicles for habits and routines. *Journal of Evolutionary Economics*, v. 14, p. 281–307, 2004.

HOLM, J. R. Applying Price's equation in empirical research. Paper presented at LEM Seminar Series. 2010.

HOLMSTROM, L. The accuracy and the computational complexity of a multivariate binned kernel density estimator. *Journal of Multivariate Analysis*, v. 72, n. 2, p. 264–309, 2000. ISSN 0047-259X. Available at: <http://www.sciencedirect.com/science/article/pii/S0047259X99918638>.

HOPENHAYN, H. Entry, exit and firm dynamics in long run equilibrium. *Econometrica*, v. 60, p. 1127–1150, 1992.

HYMER, S.; PASHIGIAN, P. Firm size and rate of growth. *Journal of Political Economy*, v. 70, p. 556–569, 1962.

HYMER, S.; PASHIGIAN, P. Firm size and rate of growth: Reply. *Journal of Political Economy*, University of Chicago Press, v. 72, n. 1, p. 83–84, 1964. ISSN 00223808, 1537534X. Available at: <http://www.jstor.org/stable/1828795>.

IBGE. *Pesquisa Industrial Anual: empresa*. Rio de Janeiro: IBGE, 2013. v. 32(1). Série Relatórios Metodológicos. Coordenação de Indústria. ISSN 0100-5138.

IJIRI, Y.; SIMON, H. A. Effects of mergers and acquisitions on business firm concentration. *Journal of Political Economy*, University of Chicago Press, v. 79, n. 2, p. 314–322, 1971. ISSN 00223808, 1537534X. Available at: <http://www.jstor.org/stable/1832112>.

IJIRI, Y.; SIMON, H. A. Interpretations of departures from the Pareto curve firm-size distributions. *Journal of Political Economy*, University of Chicago Press, v. 82, n. 2, p. 315–331, 1974. ISSN 00223808, 1537534X. Available at: <http://www.jstor.org/stable/1831181>.

IJIRI, Y.; SIMON, H. A. *Skew Distributions and the Sizes of Business Firms*. [S.l.]: North-Holland Publishing Company, 1977.

JOVANOVIC, B. Selection and the evolution of industry. *Econometrica*, v. 50, p. 649–670, 1982.

KALDOR, N. In: ______. *Capital Accumulation and Economic Growth*. New York: St. Martin's Press, 1961. p. 177–222.

KALECKI, M. On the Gibrat distribution. *Econometrica*, v. 13, n. 2, p. 161–170, April 1945.

KNUDSEN, T. General selection theory and economic evolution: The Price equation and the replicator/interactor distinction. *Journal of Economic Methodology*, v. 11, n. 2, p. 147–173, 2004.

KOMSTA, L.; NOVOMESTKY, F. *moments: Moments, cumulants, skewness, kurtosis and related tests*. [S.l.], 2015. R package version 0.14. Available at: <https://CRAN.R-project.org/package=moments>.

KRUGMAN, P. Confronting the mystery of urban hierarchy. *Journal of the Japanese and International Economies*, v. 10, n. 4, p. 399–418, 1996. ISSN 0889-1583. Available at: <http://www.sciencedirect.com/science/article/pii/S0889158396900234>.

KRUGMAN, P. R. *The Self-Organizing Economy.* Cambridge, Massachusetts: Blackwell Publishers., 1996. ISBN 1557866996.

L. Amaral; S. Buldyrev; S. Havlin; H. Leschhorn; P. Maass; M. Salinger; H. Stanley; M. Stanley. Scaling behavior in economics: I. empirical results for company growth. *J. Phys. I France*, v. 7, n. 4, p. 621–633, 1997. Available at: <https://doi.org/10.1051/jp1:1997180>.

LOTTI, F.; SANTARELLI, E.; VIVARELLI, M. The relationship between size and growth: the case of Italian newborn firms. *Applied Economics Letters*, v. 8, p. 451–454, 2001.

LOTTI, F.; SANTARELLI, E.; VIVARELLI, M. Does Gibrat's law hold among young, small firms? *Journal of Evolutionary Economics*, v. 13, n. 3, p. 213–235, 2003. Available at: <http://EconPapers.repec.org/RePEc:spr:joevec:v:13:y:2003:i:3:p:213-235>.

LUNA, I.; HIRATUKA, C.; FRACALANZA, P.; LUZ, M. In: _____. *Produtividade no Brasil : desempenho e determinantes.* Brasília: ABDI - IPEA, 2015. ch. Decomposição da evolução da produtividade na indústria e nos serviços no Brasil no período recente a partir de uma ótica evolucionária.

LUTTMER, E. G. Selection, growth, and the size distribution of firms. *The Quarterly Journal of Economics*, v. 122, n. 3, p. 1103–1144, 2007.

M., A.; NERIINGER, E. A. Testing Gibrat's law for young firms - empirical results for west germany. *Small Business Economics*, v. 15, p. 1–12, 2000.

MALERBA, F. Learning by firms and incremental technical change. *The Economic Journal*, [Royal Economic Society, Wiley], v. 102, n. 413, p. 845–859, 1992. ISSN 00130133, 14680297. Available at: <http://www.jstor.org/stable/2234581>.

MANDELBROT, B. The variation of certain speculative prices. *The Journal of Business*, University of Chicago Press, v. 36, n. 4, p. 394–419, 1963. ISSN 00219398, 15375374. Available at: <http://www.jstor.org/stable/2350970>.

MANSFIELD, E. Entry, Gibrat's law, innovation, and the growth of firms. *American Economic Review*, v. 52, p. 1023–1051, 1962. Available at: <http://EconPapers.repec.org/RePEc:kap:revind:v:24:y:2004:i:3:p:301-324>.

MARSILI, O. Technology and the size distribution of firms: Evidence from Dutch manufacturing. *Review of Industrial Organization*, Springer, v. 27, n. 4, p. 303–328, 2005. ISSN 0889938X, 15737160. Available at: <http://www.jstor.org/stable/41799265>.

MATHEW, N. Drivers of firm growth: micro-evidence from Indian manufacturing. *Journal of Evolutionary Economics*, p. 1–27, 2017. ISSN 1432-1386. Available at: <http://dx.doi.org/10.1007/s00191-017-0492-x>.

METCALFE, J. S. Competition, Fisher's principle and increasing returns in the selection process. *Journal of Evolutionary Economics*, v. 4, n. 4, p. 327–346, 1994.

METCALFE, J. S.; RAMLOGAN, R. Creative destruction and the measurement of productivity change. *Revue de l'OFCE*, v. 97, n. 5, p. 373–397, 2006.

METCALFE, S. J. *Evolutionary Economics and Creative Destruction*. Routledge: London, 1998.

MEYER, D.; DIMITRIADOU, E.; HORNIK, K.; WEINGESSEL, A.; LEISCH, F. *e1071: Misc Functions of the Department of Statistics, Probability Theory Group (Formerly: E1071), TU Wien*. [S.l.], 2017. R package version 1.6-8. Available at: <https://CRAN.R-project.org/package=e1071>.

MüLLER, K.; WICKHAM, H. *tibble: Simple Data Frames*. [S.l.], 2017. R package version 1.3.4. Available at: <https://CRAN.R-project.org/package=tibble>.

NEGRI, F. D.; CAVALCANTE, L. R. (Ed.). *Produtividade no Brasil: desempenho e determinantes*. Brasília: IPEA, 2014. v. 1. Desempenho.

NEGRI, F. D.; CAVALCANTE, L. R. (Ed.). *Produtividade no Brasil: desempenho e determinantes*. Brasília: IPEA, 2015. v. 2. Determinantes.

NELSON, R. R.; WINTER, S. G. *An evolutionary theory of economic change*. Cambridge, MA: Harvard University Press, 1982.

NEUWIRTH, E. *RColorBrewer: ColorBrewer Palettes*. [S.l.], 2014. R package version 1.1-2. Available at: <https://CRAN.R-project.org/package=RColorBrewer>.

NEWMAN, M. Power laws, Pareto distributions and Zipf's law. *Contemporary Physics*, Taylor and Francis, v. 46, n. 5, p. 323–351, 2005. Available at: <https://doi.org/10.1080/00107510500052444>.

NOGUEIRA, M.; INFANTE, R.; MUSSI, C. In: _____. *Produtividade no Brasil: desempenho e determinantes – Vol. 1 - Desempenho*. Brasília: ABD, 2014. ch. Produtividade Do Trabalho E Heterogeneidade Estrutural No Brasil Contemporâneo.

NURISSO, G.; PRESCOTT, E. Origins of too-big-to-fail policy. *Federal Reserve Bank of Cleveland, Working Paper*, n. 17-10, 2017.

OLLEY, S.; PAKES, A. The dynamics of productivity in the telecommunications equipment industry. *Econometrica*, v. 64, p. 1263–1297, 1996.

PARETO, V. *Cours d'Economie Politique*. Geneva: Droz, 1896.

PENROSE, E. *The Theory of the Growth of the Firm*. New York: Wiley, 1959.

PETRIN, A.; LEVINSOHN, J. Measuring aggregate productivity growth using plant-level data. *The RAND Journal of Economics*, v. 43, p. 705–725, 2012.

PRAIS, S. *The Evolution of Giant Firms in Britain*. London: Cambridge University Press., 1976.

PREBISCH, R. *Capitalismo Periférico: Crisis y Transformación*. Mexico: Fondo de Cultura Económica., 1981.

PRICE, G. R. Selection and covariance. *Nature, Lond.*, v. 227, p. 520–521, 1970.

PRICE, G. R. Fisher's 'fundamental theorem' made clear. *Ann. hum. Genet.*, v. 36, p. 129–140, 1972.

PRICE, G. R. The nature of selection. *Journal of Theoretical Biology*, v. 175, n. 3, p. 389–396, 1995.

R Core Team. *R: A Language and Environment for Statistical Computing*. Vienna, Austria, 2017. Available at: <https://www.R-project.org/>.

REID, G. Early life-cycle behaviour of micro-firms in Scotland. *Small Business Economics*, Springer, v. 7, n. 2, p. 89–95, 1995. ISSN 0921898X, 15730913. Available at: <http://www.jstor.org/stable/40228831>.

ROBINSON, D. *broom: Convert Statistical Analysis Objects into Tidy Data Frames.* [S.l.], 2017. R package version 0.4.2. Available at: <https://CRAN.R-project.org/package=broom>.

SAIN, S. R. Multivariate locally adaptive density estimation. *Comput. Stat. Data Anal.*, Elsevier Science Publishers B. V., Amsterdam, The Netherlands, The Netherlands, v. 39, n. 2, p. 165–186, apr. 2002. ISSN 0167-9473. Available at: <http://dx.doi.org/10.1016/S0167-9473(01)00053-6>.

SALTER, W. *Productivity and Technical Change*. 2nd edition. ed. Cambridge: Cambridge University Press, 1966.

SAMUELS, J. M. Size and the growth of firms. *Review of Economic Studies*, v. 32, n. 2, p. 105–112, 1965. Available at: <http://EconPapers.repec.org/RePEc:oup:restud:v:32:y:1965:i:2:p:105-112.>

SANTOLERI, P.; STUMPO, G. Microempresas y PYMES en América Latina: características de las firmas y políticas de apoyo. CEPAL - Comisión Económica para América Latina y el Caribe. To be published soon. 2016.

SCHUMPETER, J. A. Vilfredo Pareto (1848–1923). *The Quarterly Journal of Economics*, v. 63, n. 2, p. 147–173, 1949. Available at: <+http://dx.doi.org/10.2307/1883096>.

SCOTT, D. *Multivariate Density Estimation: Theory, Practice, and Visualization.* Second edition. Hoboken, New Jersey: John Wiley & Sons, Inc., 2015.

SEBRAE. *Participação das Micro e Pequenas Empresas na Economia Brasileira.* [S.l.]: Serviço Brasileiro de Apoio às Micro e Pequenas Empresas, 2014.

SILVERBERG, G.; DOSI, G.; ORSENIGO, L. Innovation, diversity and diffusion: a self-organisation model. *Economic Journal*, v. 98, p. 1032–1054, 1988.

SILVERBERG, G.; VERSPAGEN, B. An evolutionary model of long term cyclical variations of catching up and falling behind. *Journal of Evolutionary Economics*, v. 5, n. 3, p. 209–227, 1995.

SILVERMAN, B. *Density Estimation for Statistics and Data Analysis.* London: Chapman and Hall, 1986.

SIMON, H. On a class of skew distribution functions. *Biometrika*, v. 44, p. 425–440, 1955.

SIMON, H. A. Comment: Firm size and rate of growth. *Journal of Political Economy*, University of Chicago Press, v. 72, n. 1, p. 81–82, 1964. ISSN 00223808, 1537534X. Available at: <http://www.jstor.org/stable/1828794>.

SIMON, H. A.; BONINI, C. P. The size distribution of business firms. *American Economic Review*, v. 48, p. 607–617, 1958.

SINGH, A.; WHITTINGTON, G. The size and growth of firms. *Review of Economic Studies*, John Wiley and Sons, v. 42, 04 1975.

SOARES, E.; CASTILHO, M. *O Impacto das Importações Chinesas na Indústria Brasileira nos Anos 2000.* [S.l.], 2016. Available at: <https://EconPapers.repec.org/RePEc:anp:en2014:152>.

SOLOW, R. *We'd better watch out.* 1987. New York Times Book Review, p. 36.

SQUEFF, G.; NOGUEIRA, M. In: _____. *Por um desenvolvimento inclusivo: O caso do Brasil.* Santiago: Nacoes Unidas, 2015. ch. A heterogeneidade estrutural no Brasil de 1950 a 2009.

STANLEY, H.; GABAIX, X.; GOPIKRISHNAN, P.; PLEROU, V. Economic fluctuations and statistical physics: Quantifying extremely rare and less rare events in finance. *Physica A: Statistical Mechanics and its Applications*, v. 382, n. 1, p. 286–301, 2007. ISSN 0378-4371. Applications of Physics in Financial Analysis. Available at: <http://www.sciencedirect.com/science/article/pii/S0378437107001586>.

STANLEY, M.; AMARAL, L.; BULDYREV, S.; HAVLIN, S.; LESCHHORN, H.; MAASS, P.; SALINGER, M.; STANLEY, H. Scaling behaviour in the growth of companies. *Nature*, v. 379, p. 804–806, 1996. Available at: <+http://dx.doi.org/10.1038/379804a0>.

STANLEY, M.; BULDYREV, S.; HAVLIN, S.; MANTEGNA, R.; SALINGER, M.; STANLEY, H. E. Zipf plots and the size distribution of firms. *Economics Letters*, v. 49, n. 4, p. 453–457, 1995. ISSN 0165-1765. Available at: <http://www.sciencedirect.com/science/article/pii/016517659500696D>.

STEINDL, J. *Random Process and the Growth of Firms: A Study of the Pareto Law.* New York: Hafner, 1965.

STURGEON, T. J. Modular production networks: a new american model of industrial organization. *Industrial and Corporate Change*, v. 11, n. 3, p. 451–496, 2002. Available at: <https://EconPapers.repec.org/RePEc:oup:indcch:v:11:y:2002:i:3:p:451-496>.

SUBBOTIN, M. On the law of frequency of errors. *Matematicheskii Sbornik*, v. 31, p. 296–301, 1923.

SUTTON, J. Gibrat's legacy. *Journal of Economic Literature*, v. 35, p. 40–59, 1997.

SWIHART, B.; LINDSEY, J. *rmutil: Utilities for Nonlinear Regression and Repeated Measurements Models.* [S.l.], 2017. R package version 1.1.0. Available at: <https://CRAN.R-project.org/package=rmutil>.

SYVERSON, C. What determines productivity? *Journal of Economic Literature*, v. 49, n. 2, p. p. 326–365, 2011.

TEECE, D. J.; PISANO, G.; SHUEN, A. Dynamic capabilities and strategic management. *Strategic Management Journal*, Wiley, v. 18, n. 7, p. 509–533, 1997. ISSN 01432095, 10970266. Available at: <http://www.jstor.org/stable/3088148>.

TSYBAKOV, A. *Introduction to Nonparametric Estimation.* New York: Springer-Verlag, 2009. (Springer Series in Statistics.). DOI: 10.1007/b13794.

TUKEY, J. W. The future of data analysis. *Ann. Math. Statist.*, The Institute of Mathematical Statistics, v. 33, n. 1, p. 1–67, 03 1962. Available at: <https://doi.org/10.1214/aoms/1177704711>.

WAND, M. P. Fast computation of multivariate kernel estimators. *Journal of Computational and Graphical Statistics*, v. 3, n. 4, p. 433–445, 1994. Available at: <http://www.tandfonline.com/doi/abs/10.1080/10618600.1994.10474656>.

WEST, G. *Scale: The Universal Laws of Growth, Innovation, Sustainability, and the Pace of Life in Organisms, Cities, Economies, and Companies*. [S.l.]: Penguin Press., 2017.

WICKHAM, H. Reshaping data with the reshape package. *Journal of Statistical Software*, v. 21, n. 12, p. 1–20, 2007. Available at: <http://www.jstatsoft.org/v21/i12/>.

WICKHAM, H. *ggplot2: Elegant Graphics for Data Analysis*. Springer-Verlag New York, 2009. ISBN 978-0-387-98140-6. Available at: <http://ggplot2.org>.

WICKHAM, H. *lazyeval: Lazy (Non-Standard) Evaluation*. [S.l.], 2017. R package version 0.2.1. Available at: <https://CRAN.R-project.org/package=lazyeval>.

WICKHAM, H. *scales: Scale Functions for Visualization*. [S.l.], 2017. R package version 0.5.0. Available at: <https://CRAN.R-project.org/package=scales>.

WICKHAM, H. *tidyverse: Easily Install and Load 'Tidyverse' Packages*. [S.l.], 2017. R package version 1.1.1. Available at: <https://CRAN.R-project.org/package=tidyverse>.

WICKHAM, H. *stringr: Simple, Consistent Wrappers for Common String Operations*. [S.l.], 2018. R package version 1.3.0. Available at: <https://CRAN.R-project.org/package=stringr>.

WICKHAM, H.; FRANCOIS, R.; HENRY, L.; MüLLER, K. *dplyr: A Grammar of Data Manipulation*. [S.l.], 2017. R package version 0.7.4. Available at: <https://CRAN.R-project.org/package=dplyr>.

WICKHAM, H.; HENRY, L. *tidyr: Easily Tidy Data with 'spread()' and 'gather()' Functions*. [S.l.], 2017. R package version 0.7.2. Available at: <https://CRAN.R-project.org/package=tidyr>.

WILKE, C. O. *cowplot: Streamlined Plot Theme and Plot Annotations for 'ggplot2'*. [S.l.], 2017. R package version 0.8.0. Available at: <https://CRAN.R-project.org/package=cowplot>.

WINTER, S. G. Schumpeterian competition under alternative technological regimes. *Journal of Economic Behavior and Organization*, v. 5, p. 287–320, 1984.

WINTER, S. G.; KANIOVSKI, Y. M.; DOSI, G. Modeling industrial dynamics with innovative entrants. *Structural Change and Economic Dynamics*, v. 11, p. 255–293, 2000.

WINTER, S. G.; KANIOVSKI, Y. M.; DOSI, G. A baseline model of industry evolution. *Journal of Evolutionary Economics*, v. 13, p. 355–383, 2003.

WUERTZ, D.; SETZ, T.; CHALABI, Y. *fBasics: Rmetrics - Markets and Basic Statistics*. [S.l.], 2017. R package version 3042.89. Available at: <https://CRAN.R-project.org/package=fBasics>.

YEE, T. W. *VGAM: Vector Generalized Linear and Additive Models*. [S.l.], 2018. R package version 1.0-5. Available at: <https://CRAN.R-project.org/package=VGAM>.

YU, X.; DOSI, G.; GRAZZI, M.; LEI, J. Inside the virtuous cycle between productivity, profitability, investment and corporate growth: An anatomy of China industrialization. *Quaderni - Working Paper DSE N. 1006*, 2015. Available at SSRN: http://ssrn.com/abstract=2605030 or http://dx.doi.org/10.2139/ssrn.2605030.

YU, X.; DOSI, G.; LEI, J.; NUVOLARI, A. Institutional change and productivity growth in China's manufacturing: the microeconomics of knowledge accumulation and creative restructuring. *Industrial and Corporate Change*, v. 24, n. 3, p. 565–602, 2015.

YULE, G. U. A mathematical theory of evolution, based on the conclusions of dr. J. C. Willis, F.R.S. *Philosophical Transactions of the Royal Society of London. Series B, Containing Papers of a Biological Character*, The Royal Society, v. 213, p. 21–87, 1925. ISSN 02643960. Available at: <http://www.jstor.org/stable/92117>.

ZEILEIS, A.; HOTHORN, T. Diagnostic checking in regression relationships. *R News*, v. 2, n. 3, p. 7–10, 2002. Available at: <https://CRAN.R-project.org/doc/Rnews/>.

ZIPF, G. *Human Behaviour and the Principle of Least Effort*. Reading, MA: Addison-Wesley, 1949.

ZOU, Y.; FONOBEROV, V. A.; FONOBEROVA, M.; MEZIC, I.; KEVREKIDIS, I. G. Model reduction for agent-based social simulation: Coarse-graining a civil violence model. *Phys. Rev. E*, American Physical Society, v. 85, p. 066106, Jun 2012. Available at: <https://link.aps.org/doi/10.1103/PhysRevE.85.066106>.